Productivity and the Welfare of Nations*

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Abstract

We show that the welfare of a country's infinitely-lived representative consumer is summarized, to a first order, by total factor productivity and by the capital stock per-capita. These variables suffice to calculate welfare changes within a country, as well as welfare differences across countries. The result holds regardless of the type of production technology and the degree of product market competition. It applies to open economies as well, if total factor productivity is constructed using domestic absorption, instead of gross domestic product, as the measure of output. It also requires that total factor productivity be constructed with prices and quantities as perceived by consumers, not firms. Thus, factor shares need to be calculated using after-tax wages and rental rates and they will typically sum to less than one. These results are used to calculate welfare gaps and growth rates in a sample of developed countries with high-quality total factor productivity and capital data. Under realistic scenarios, the U.K. and Spain had the highest growth rates of welfare during the sample period 1985-2005, but the U.S. had the highest level of welfare.

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1 Introduction

Standard models in many fields of economics posit the existence of a representative household in either a static or a dynamic setting, and then seek to relate the welfare of that household to observable aggregate data. A separate large literature examines the productivity residual defined by Solow (1957), and interprets it as a measure of technical change or policy effectiveness. Yet a third literature, often termed "development accounting," studies productivity differences across countries, and interprets them as measures of technology gaps or institutional quality. To our knowledge, no one has suggested that these three literatures are intimately related. We show that they are. We start from the standard framework of a representative household that maximizes intertemporal welfare over an infinite horizon, and use it to derive methods for comparing economic well-being over time and across countries. Our results show that under a wide range of assumptions, welfare can be measured using just two variables, productivity and capital accumulation. We take our framework to the data, and measure welfare change within countries and welfare differences across countries.

In the simplest case of a closed economy with no distortionary taxes we show that to a first-order approximation the welfare change of a representative household can be fully characterized by three objects: the expected present discounted value of total factor productivity (TFP) growth as defined by Solow, the change in expectations of the level of TFP, and the growth in the stock of capital per person. The result sounds similar to one that is often proven in the context of a competitive optimal growth model, which might lead one to ask what assumptions on technology and product market competition are required to obtain this result. The answer is, None. The result holds for all types of technology and market behavior, as long as consumers take prices as given and are not constrained in the amount they can buy or sell at those prices. Thus, for example, the same result holds whether the TFP growth is generated by exogenous technological change, as in the Ramsey-Cass-Koopmans model; by changes in the size of the economy combined with increasing returns to scale, as in the "semi-endogenous growth" models of Arrow (1962) and Jones (1995); or by externalities or public policy in fully-endogenous growth models, such as Romer (1986) or Barro (1990). As we discuss below, aggregate TFP can also change without any change in production technology in multi-sector models with heterogeneous distortions (for example, markups that differ across sectors): our results show that an increase in aggregate TFP due to reallocation would be as much of a welfare gain for the representative consumer as a change in exogenous technology with the same magnitude and persistence.

Our findings suggest a very different interpretation of TFP from the usual one. Usually one argues that TFP growth is interesting because it provides information on the change or diffusion of technology, or measures improvement in institutional quality, the returns to scale in the production function, or the markup of price over marginal cost. We show that whether all or none of these things is true, TFP is interesting for a very different reason. Using only the first-order conditions for optimization of the representative household, we can show that TFP is key to measuring welfare changes within a country and welfare differences across countries. We interpret TFP purely from

the household side, producing what one might call "the household-centric Solow residual."[1] Here we follow the intuition of Basu and Fernald (2002), and supply a general proof of their basic insight that TFP, calculated from the point of view of the consumer, is relevant for welfare.

The intuition for our result comes from noting that TFP growth is output growth minus share-weighted input growth. The representative household receives all output, which *ceteris paribus* increases its welfare. But at the same time it supplies some inputs: labor input, which reduces leisure, and capital input, which involves deferring consumption (and perhaps losing some capital to depreciation). The household measures the cost of the inputs supplied relative to the output gained by real factor prices–the real wage and the real rental rate of capital. TFP also subtracts inputs supplied from output gained, and uses exactly the same prices to construct the input shares. The welfare result holds in a very general setting because relative prices measure the consumer's marginal rate of substitution even in many situations when they do not measure the economy's marginal rate of transformation–for example, if there are externalities, increasing returns or imperfect competition.

This intuition suggests that in cases where prices faced by households differ from those facing firms, it is the former that matter for welfare. We show that this intuition is correct, and here our household-centric Solow residual substantially differs from Solow's original measure, which uses the prices faced by firms. Proportional taxes are an important source of price wedges in actual economies. We show that the shares in the household-centric Solow residual need to be constructed using the factor prices faced by households. Since marginal income tax rates and rates of value-added taxation can be substantial, especially in rich countries, this modification is quantitatively important, as we show in empirical implementations of our results.

We then move to showing analogous results for open economies. Here we show that our previous results need to be modified substantially if we construct TFP using the standard output measure, real GDP. To the three terms discussed above we need to add the present discounted value of expected changes in the terms of trade, the present discounted value of expected changes in the rate of return on foreign assets, and the growth rate of net foreign asset holdings. Intuitively, both the terms of trade and the rate of return on foreign assets affect the consumer's ability to obtain welfare-relevant consumption and investment for a given level of factor supply. Holdings of net foreign assets are analogous to domestic physical capital in that both can be transformed into consumption at a future date.

While these results connect to and extend the existing literature, as we discuss below, they are difficult to take to the data. It is very hard to get good measures of changes in asset holdings by country for a large sample of countries.[2] Furthermore, measuring asset returns in a comparable way across countries would require us to adjust for differences in the risk of country portfolios, which is a formidable undertaking. Fortunately, we are able to show that these difficulties disappear if we

[1] The term is due to Miles Kimball.

[2] The important work of Lane and Milesi-Ferretti (2001, 2007) has shed much light on this subject, but the measurement errors that are inevitable in constructing national asset stocks lead to very noisy estimates of net asset growth rates.

switch to using real absorption rather than GDP as the measure of output.[3] In this case, exactly the same three terms that summarize welfare in the closed economy are also sufficient statistics in the open economy. Thus, our approach using the household-centric productivity residual can be used empirically to measure welfare change in ways that are invariant to the degree of openness of the economy.

These results pertain to the evolution of welfare in individual economies over time. The indexes we obtain are not comparable across countries. Thus it is natural to ask whether our methods shed any light on a pressing and long-standing question, the measurement of relative welfare across countries using a method firmly grounded in economic theory. It turns out that they do. Perhaps our most striking finding is the result that we can use data on cross-country differences in TFP and capital intensity, long the staples of discussion in the development and growth literatures, to measure differences in welfare across countries. More precisely, we show that productivity and the capital stock suffice to calculate differences in welfare across countries, with both variables computed as log level deviations from a reference country.

To understand this result, it helps to deepen the intuition offered above. Our analysis is based on a dynamic application of the envelope theorem, and it shows that the welfare of a representative agent depends to a first order on the expected time paths of the variables that the agent takes as exogenous. In a dynamic growth context, these variables are the prices for factors the household supplies (labor and capital), the prices for the goods it purchases (consumption and investment), and beginning-of-period household assets, which are predetermined state variables and equal to the capital stock in a closed economy. Apart from this last term, the household's welfare depends on the time paths of *prices*, which are exogenous to the household. Thus, the TFP that is directly relevant for household welfare is actually the *dual* Solow residual. We use the national income accounts identity to transform the dual residual into the familiar primal Solow residual.

Our cross-country welfare result comes from using the link between welfare and exogenous prices implied by economic theory to ask how much an individual's welfare would differ if he faced the sequence of prices, not of his own country, but of some other country. We can perform the thought experiment of having a US consumer face the expected time paths of all goods and factor prices in, say, France, and also endow him with beginning-of-period French assets rather than US assets. The difference between the resulting level of welfare and the welfare of remaining in the US measures the gain or loss to a US consumer of being moved to France. Note that our welfare comparisons are from a definite point of view—in this example, from the view of a US consumer. In principle, the result could be different if the USA-France comparison is made by a French consumer, with different preferences over consumption and leisure. Fortunately, our empirical results are qualitatively unchanged and quantitatively little affected by the choice of the "reference country" used for these welfare comparisons.

The same insights that apply to the time series are relevant for the cross section: TFP needs to be defined using the prices perceived by households, and if the economy is open then other

[3] We are indebted to Mikhail Dmitriev for pointing out this result.

terms become relevant. Thus, tax rates, terms of trade, and foreign asset holdings also matter for cross-country welfare comparisons. As before, we can reduce the measurement complications enormously by using absorption rather than GDP as the definition of output in our household-centric TFP measure.

These results show that we can use readily-available national income accounts data to perform interesting welfare comparisons highlighting the role of productivity. We illustrate our methods using data for several industrialized countries for which high-quality data are available: Canada, France, Italy, Japan, Spain, the United Kingdom and the United States. We show the importance of fiscal considerations in constructing measures of welfare change over time. For example, if we assume that government spending is wasteful and taxes are lump-sum, the UK has the largest welfare gain among our group of countries over our sample period, 1985-2005, while Spain lags far behind due to its low TFP growth rate. Indeed, the US, a much richer country, has faster welfare growth than Spain under these assumptions. Allowing for distortionary taxation and assuming that government expenditure is not wasteful, Spain has the highest welfare growth among all countries, with the UK a shade behind, and the US much further back.

However these welfare growth rates are country-specific indexes, and cannot be used to compare welfare across countries. We next apply our methodology to cross country-comparisons and show how these relative welfare levels evolve over time. In our benchmark case of optimal government spending and distortionary taxation, the US is the welfare leader throughout our sample period. At the start of our sample, we find that France and the UK are closest to the US in terms of welfare, with France having a slight advantage over the UK. By the end of the sample, France and most of the other economies fall further behind the US in terms of welfare levels, with the two exceptions being Spain and the UK. Spain converges towards the US level of welfare in the first several years of the sample, and then holds steady at a constant percent gap. The UK, by contrast, converges towards the US at a relatively constant rate, and by 2005 is within a few percent of the US level of per-capita welfare.

The paper is structured as follows. The next section presents our analytical framework, and uses it to derive results on the measurement of welfare within single economies and on welfare comparisons across countries. (Full derivations of the results in Section 2 are presented in an appendix.) We present a number of extensions to our basic framework in Section 3, allowing for multiple types of goods and factors, distortionary taxes, government expenditure and an open economy. We then take the enhanced framework to the data, and discuss empirical results in Section 4. We discuss relations of our work to several distinct literatures in Section 5. Finally, we conclude by summarizing our findings and suggesting fruitful avenues for future research.

2 The Productivity Residual and Welfare

Both intuition and formal empirical work link TFP growth to increases in the standard of living, at least as measured by GDP per-capita.[4] The usual justification for studying the Solow productivity residual is that, under perfect competition and constant returns to scale, it measures technological change, which contributes to GDP growth, one major determinant of welfare. Thus, the usual connection between Solow's residual and welfare is a round-about one. Furthermore, this intuition suggests that we should not care about the Solow residual in an economy with non-competitive output markets, non-constant returns to scale, and possibly other distortions where the Solow residual is no longer a good measure of technological progress. We show that the link between Solow's residual and welfare is immediate and solid, even when the residual does not measure technical change. Here we build on the intuition of Basu and Fernald (2002) and derive rigorously the relationship between a modified version of the productivity residual and the intertemporal utility of the representative household. The fundamental result we obtain is that, to a first-order approximation, utility reflects the present discounted value of productivity residuals (plus the initial stock of capital).

Our results are complementary to those in Solow's classic (1957) paper. Solow established that if there was an aggregate production function then his index measured its rate of change. We now show that under a very different set of assumptions, which are disjoint from Solow's, the familiar TFP index is also the key component of an intertemporal welfare measure. The results are parallel to one another. Solow did not need to assume anything about the consumer side of the economy to give a technical interpretation to his index, but he had to make assumptions about technology and firm behavior. We do not need to assume anything about the firm side (which includes technology, but also firm behavior and industrial organization) in order to give a welfare interpretation, but we do need to assume the existence of a representative consumer.[5] Which result is more useful depends on the application, and the trade-off that one is willing to make between having a result that is very general on the consumer side but requires very precise assumptions on technology and firm behavior, and a result that is just the opposite.

2.1 Measuring welfare changes over time

We begin by assuming the familiar objective function for a representative household that maximizes intertemporal utility. In a growth context one often neglects the dependence of welfare on leisure, but the work of Nordhaus and Tobin (1972) suggests that this omission is not innocuous (see the discussion in Section 5). Thus, we assume the household derives utility from both consumption and leisure:

[4] For a review of the literature linking TFP to GDP per worker, in both levels and growth rates, see Weil (2008).

[5] At a technical level, both results assume the existence of a potential function (Hulten, 1973), and show that TFP is the rate of change of that function.

$$W_t = E_t \sum_{s=0}^{\infty} \frac{1}{(1+\rho)^s} \frac{N_{t+s}}{H} \frac{1}{1-\sigma} {C_{t+s}}^{1-\sigma} \nu(\overline{L} - L_{t+s}) \tag{1}$$

where W_t denotes the total welfare of the household, C_t is the per-capita consumption at time t, L_t are per-capita hours of work and $\overline{L}$ is the per-capita time endowment. N_t is population and H is the number of households, assumed to be fixed and normalized to one from now on. Population grows at a constant rate n. To ensure the existence of a well-defined steady-state in which hours of work are constant while consumption and the real wage share a common trend, we assume that the utility function has the King, Plosser and Rebelo (1988) form with $\sigma > 0$ and $\nu(.) > 0$.[6] The budget constraint facing the representative consumer and the capital accumulation equation are respectively:

$$P_t^I K_t N_t + B_t N_t = (1-\delta) P_t^I K_{t-1} N_{t-1} + \left(1 + i_t^B\right) B_{t-1} N_{t-1} + P_t^L L_t N_t + P_t^K K_{t-1} N_{t-1} + \Pi_t N_t - P_t^C C_t N_t \tag{2}$$

and

$$K_t N_t = (1-\delta) K_{t-1} N_{t-1} + I_t N_t \tag{3}$$

K_t, B_t and I_t denote per-capita capital, bonds and investment; P_t^K, P_t^L, P_t^C and P_t^I denote, respectively, the user cost of capital, the hourly wage, the price of consumption goods and of new capital goods; $\left(1 + i_t^B\right)$ is the nominal interest rate and Π_t denotes per-capita profits, which are paid lump-sum from firms to consumers. Assume for now that the economy is closed and there is no government, which implies that in equilibrium $B_t = 0$. (We derive analogous results for the open economy with capital mobility and unbalanced trade ($B_t \neq 0$) in Section 3.4, and extend the results in this section to allow for government expenditure, distortionary or lump-sum taxes, and government bond issuance in Sections 3.1-3.2.)

Define "equivalent consumption" per person, denoted by C_t^*, as the level of consumption per-capita at time t that, if growing at the steady-state rate g from t onward, with leisure set at its steady-state level, delivers the same intertemporal utility per-capita as the actual stream of consumption and leisure. More precisely, C_t^* satisfies:

$$\begin{aligned} \frac{W_t}{N_t} &= V_t = \sum_{s=0}^{\infty} \frac{(1+n)^s}{(1+\rho)^s} \left(C_t^* (1+g)^s\right)^{1-\sigma} \nu(\overline{L} - L)) \\ &= \frac{1}{(1-\sigma)(1-\beta)} C_t^{*1-\sigma} \nu(\overline{L} - L) \end{aligned} \tag{4}$$

where V_t denotes per-capita utility and $\beta = \frac{(1+n)(1+g)^{1-\sigma}}{(1+\rho)}$ is the discount rate in the problem reformulated in terms of stationary variables to allow for steady-state growth.

[6] If $\sigma = 1$, then the utility function must be $U(C_1, .., C_G; \overline{L} - L) = \log(C) - \nu(\overline{L} - L)$. See King, Plosser and Rebelo (1988).

We will measure welfare changes over time in terms of equivalent consumption per-capita and relate them to observable economic variables.

First we define a few of the key variables used in our analysis. Consider a modified definition of the Solow productivity residual:

$$\Delta \log PR_{t+s} \equiv \Delta \log Y_{t+s} - s_L \Delta \log L_{t+s} - s_K \Delta \log K_{t+s-1} \tag{5}$$

where $\Delta \log Y_t = s_C \Delta \log C_t + s_I \Delta \log I_t$. $\Delta \log Y_t$ is a Divisia index of per-capita GDP growth, where demand components are aggregated using constant steady-state shares. s_C and s_I denote the steady-state values of $s_{C,t} = \frac{P_t^C C_t}{p_t^Y Y_t}$and $s_{I,t} = \frac{P_t^I I_t}{P_t^Y Y_t}$ respectively, and $P_t^Y Y_t$ represents per-capita GDP in current prices.[7] Distributional shares are also defined as the steady-state values, s_L and s_K, of $s_{L,t} \equiv \frac{P_t^L L_t}{P_t^Y Y_t}$ and $s_{K,t} \equiv \frac{P_t^K K_{t-1} N_{t-1}}{P_t^Y Y_t N_t}$ (note that the household receives remuneration on the capital stock held at the end of the last period). We use the word "modified" in describing the productivity residual for three reasons. First, we do not assume that the distributional shares of capital and labor add to one, as they would if there were zero economic profits and no distortionary taxes.[8] Second, all shares are calculated at their steady-state values and, hence, are not time varying, which is sometimes assumed when calculating the residual.[9] Third, the residual is stated in terms of per-capita rather than aggregate variables, although it should be noted that Solow himself defined the residual on a per-capita basis (1957, equation 2a). Correspondingly, define the log level productivity residual as:

$$\log PR_{t+s} \equiv s_C \log C_{t+s} + s_I \log I_{t+s} - s_L \log L_{t+s} - s_K \log K_{t+s-1} \tag{6}$$

The prices in the budget constraint, equation (2), are defined in nominal terms. It will often be easier to work with relative prices, and disregard complications that arise from price inflation. Taking the purchase price of new capital goods, P_t^I, as numeraire, define the following relative prices: $p_t^K = \frac{P_t^K}{P_t^I}$, $p_t^L = \frac{P_t^L}{P_t^I}$ and $p_t^C = \frac{P_t^C}{P_t^I}$. Real per-capita profits are defined as $\pi_t = \frac{\Pi_t}{P_t^I}$. Our approximations are taken around a steady-state path where the first three relative prices are constant and the wage p^L grows at rate g, as in standard one-sector models of economic growth. We also assume that all per-capita quantity variables other than labor hours (for example Y_t, C_t, I_t, etc.) grow at a common rate g in the steady-state. Note these assumptions imply that all of the shares we have defined above are constant in the steady-state and so is the capital output ratio, whose nominal steady-state value will be denoted by $\frac{P^I K}{P^Y Y}$.[10]

[7] For now we set government expenditure to zero, and introduce it in our extension in Section 3.2. Our definition of GDP departs slightly from convention, as value added is usually calculated using time-varying shares. The two definitions coincide to a first-order approximation.

[8] Zero profits are guaranteed in the benchmark case with perfect competition and constant returns to scale, but can also arise with imperfect competition and increasing returns to scale—as long as there is free entry—as in the standard Chamberlinian model of imperfect competition.

[9] Rotemberg and Woodford (1991) argue that in a consistent first-order log-linearization of the production function the shares of capital and labor should be taken to be constant, and Solow's (1957) use of time-varying shares amounts to keeping some second-order terms while ignoring others.

[10] We conjecture that all our results could be proved in the household environment corresponding to a two-sector

Under these assumptions we can show that welfare changes, as measured by equivalent consumption, C_t^*, are, to a first-order approximation, a linear function of the expectation of present and future total factor productivity growth (and its revision), and of the initial capital stock. This first key result is summarized in:

Proposition 1 *Assume that the representative household in a closed economy with no government maximizes (1) subject to (2), taking prices, profits and interest rates as exogenously given. Assume also that population grows at a constant rate n, and the wage and all per-capita quantities other than labor hours grow at rate g in the steady-state. To a first-order approximation, the growth rate of equivalent consumption can be written as:*

$$\Delta \log C_t^* = \frac{(1-\beta)}{s_c}\left[E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} + \sum_{s=0}^{\infty} \beta^s \Delta E_t \log PR_{t+s} + \frac{1}{\beta}\left(\frac{P^I K}{P^Y Y}\right)\Delta \log K_{t-1}\right] \quad (7)$$

Proof. Proofs of all propositions and extensions are collected in the Appendix. ■

Proposition 1 implies that the expected present discounted value of current and future Solow productivity residuals, together with the change in the initial stock of capital per-capita, is a sufficient statistic for the welfare of a representative consumer (where we measure welfare as the log change in equivalent consumption). The term $\Delta E_t \log PR_{t+s} = E_t \log PR_{t+s} - E_{t-1} \log PR_{t+s}$ represents the revision in expectations of the log level of the productivity residual, based on the new information received between $t-1$ and t. Note that the expectation revision terms in the second summation will reduce to a linear combination of the innovations in the stochastic shocks affecting the economy at time t. Moreover, if we assume that the modified log level productivity residual follows a univariate autoregressive process, then only the innovation of such a process matters for the expectation revision, and the first summation is simply function of current and past values of productivity.

Since we have not made any assumptions about production technology and firm behavior, the productivity terms may or may not measure technical change. For example, Solow's residual does not measure technical change in economies where firms have market power, or produce with increasing returns to scale, or where there are Marshallian externalities. Even in these cases, Proposition 1 shows that productivity and the capital stock are jointly a sufficient statistic for welfare. Finally, as we show in Section 3, this basic result can be proved in much more general environments—for example, in an open economy, with government expenditure and debt, distortionary taxes, multiple consumption goods, and many types of labor.

While the proof of the proposition requires somewhat complex notation and algebra, in the remainder of this sub-section we shall try to convey the economic reasoning for the result by considering the much simpler case of an economy with a zero steady-state growth rate ($g = 0$). (Of course, the formal proof of Proposition 1 allows for $g > 0$.) We begin by taking a first-order

growth model as laid out, for example, in Whelan (2003)—assuming that the steady-state shares are also constant, as in Whelan's setup.

approximation to the *level* of utility of the household (normalized by population).[11] We then use the household's first-order conditions for optimality to obtain:

$$\frac{(V_t - V)}{\lambda p^Y Y} = E_t \sum_{s=0}^{\infty} \beta^s \left[s_L \widehat{p^L_{t+s}} + s_K \widehat{p^K_{t+s}} + s_\pi \, \widehat{\pi}_t - s_C \widehat{p^C_{t+s}} \right] + \frac{1}{\beta} \left(\frac{P^I K}{P^Y Y} \right) \widehat{K_{t-1}} \tag{8}$$

Hatted variables denote log deviations from the steady-state ($\widehat{x}_t = \log x_t - \log x$). Variables without time subscripts denote steady-state values. Since $g = 0$, $\beta = \frac{1+n}{1+\rho}$. λ is the Lagrange multiplier associated with the budget constraint expressed, like utility, in per-capita terms. Equation (8) follows almost directly from the Envelope Theorem. An atomistic household maximizes taking as given the sequences of current and expected future prices, lump-sum transfers (Π_t), and predetermined variables (in our environment, just K_{t-1}). Thus, only fluctuations in these objects affect welfare to a first order. The Envelope Theorem plus a bit of algebra shows that each change in exogenous prices or profits needs to be multiplied by its corresponding share to derive its effect on welfare—for example, the larger is s_C the more the consumer suffers from a rise in the relative price of consumption goods. (It may appear that the investment price is missing, but since we normalized the relative price of investment goods to 1 it never changes.) The terms within the summation can be thought of as the dual version of the productivity residual, as we will show shortly.

The left hand side of the equation has an interesting interpretation: It is the money value of the deviation of per-person utility from its steady-state level, expressed as a fraction of steady-state GDP per person. To understand this interpretation, consider the units. The numerator is in "utils," which we divide by λ, which has units of utils/investment good (since investment goods are our numeraire). The division gives us the deviation of utility from its steady-state value measured in units of investment goods in the numerator, divided by the real value of per-capita GDP, also stated in terms of investment goods (recall that p^Y is a relative price). However, we find it more convenient and intuitive to express the left hand side of (8) in terms of equivalent consumption. Using the definition in (4) and taking a first order approximation of $V_t - V$ in terms of log C_t^* we obtain:

$$\frac{(V_t - V)}{\lambda p^Y Y} = \frac{s_C}{(1-\beta)} (\log C_t^* - \log C) \tag{9}$$

where we have used the fact that in the steady-state $C^* = C$ and $U_C = \lambda p^c$.

The right hand side of (8) is written as a function of the log deviation from the steady-state of prices, profits and the initial capital stock. Our results can also be presented using the familiar (primal) productivity residual rather than stating them in terms of prices and transfers, as in equation (8). However, if one uses a consistent data set, there is literally no difference between the two. One can show, using the per-capita version of the household budget constraint (2) and the capital accumulation equation (3), that the following relationship must hold at all points in time:

[11] We approximate level of V rather than its log because $V < 0$ if $\sigma > 1$.

$$s_L\widehat{p^L_{t+s}} + s_K\widehat{p^K_{t+s}} + s_\pi\widehat{\pi}_t - s_C\widehat{p^C_{t+s}} = s_C\widehat{C}_{t+s} + s_I\widehat{I_{t+s}} - s_L\widehat{L}_{t+s} - s_K\widehat{K_{t+s-1}}. \tag{10}$$

Since the budget constraint of the representative household is just the national income accounts identity in per-capita terms, equation (10) says that in any data set where national income accounting conventions are enforced, the primal productivity residual identically equals the dual productivity residual; see, for example, Barro and Sala-i-Martin (2004, section 10.2). Thus we can express our results in either form, but using the dual result would require us to provide an empirical measure of lump-sum transfers, which is not needed for results based on the primal residual. Mostly for this reason, we work with the primal.

Using (9) and (10) in (8) we can write:

$$(\log C_t^* - \log C) = \frac{(1-\beta)}{s_C}\left[E_t\sum_{s=0}^{\infty}\beta^s\widehat{PR}_{t+s} + \frac{1}{\beta}\left(\frac{P^I K}{P^Y Y}\right)\widehat{K_{t-1}}\right] \tag{11}$$

where now the log deviation of consumption is expressed as a function of the log deviation from steady-state of the productivity residual, defined in equation (6). Taking differences of equation (11) and using the definition of the Solow residual in (5) gives the statement of Proposition 1, whose proof we have just sketched for the case of $g = 0$.

2.2 Implications for Cross Country Analysis

Proposition 1 pertains to the evolution of welfare in individual economies over time. The indexes we obtain are not comparable across countries. Thus it is natural to ask whether our methods shed any light on a pressing and long-standing question, the measurement of relative welfare across countries using a method firmly grounded in economic theory. It turns out that they do. Perhaps our most striking finding is that we can use data on cross-country differences in TFP and capital intensity, long the staples of discussion in the development and growth literatures, to measure differences in welfare across countries. More precisely, we show that productivity and the capital stock suffice to calculate differences in welfare across countries, with both variables computed as log level deviations from a reference country.

Welfare comparisons across countries have been investigated recently by Jones and Klenow (2010), who focus on a point-in-time comparison of single-period flow utility. By comparison, we focus on intertemporal (lifetime) utility, and show how out-of-steady-state dynamics are related to capital accumulation and productivity. Our method emphasizes the dynamic welfare effects resulting from the fact that lower consumption or leisure today may raise capital accumulation and support greater consumption in the future.[12] In settings such as the Ramsey model where measured productivity reflects only underlying technology, our method relates welfare to its deeper causal determinants: an exogenous state variable (productivity) and an endogenous but predetermined

[12] We do not, however, allow for cross country differences in life expectancy or in inequality as in Jones and Klenow (2010).

state variable (the capital stock). Of course, one of our novel results is that measured productivity and the initial capital stock continue to summarize welfare even outside the Ramsey setting.

A comparison of welfare across countries requires either assuming that their respective representative agents possess the same utility function, or making the comparison from the perspective of the representative agent in a reference country. We follow the latter interpretation in the exposition and consider the thought experiment of a household from a reference country j facing the prices, per-capita profits and initial capital stock of country i instead of those in country j. We then study the difference in the utility of a representative member of the household and, as in the within-country case, we conduct the comparison by using the concept of equivalent consumption. In this context for the representative agent of the reference country j living in country i, equivalent per-capita consumption, $\widetilde{C}_t^{*,i}$ satisfies:

$$\widetilde{V}_t^i = \frac{1}{\left(1-\sigma^j\right)\left(1-\beta^j\right)}\left(\widetilde{C}_t^{*,i}\right)^{1-\sigma}\nu(\overline{L}-L^j) \tag{12}$$

where $\widetilde{V}_t^i$ denotes per-capita utility of the individual from country j, facing country i's relative prices, per-capita profits and per-capita initial capital stock. Note that $\widetilde{C}_t^{*,i}$ is defined for a constant level of leisure fixed at country j's steady-state level. We will use V_t^j and $C_t^{*,i}$ to denote per-capita utility and equivalent consumption of the individual of country j living in country j.

Proposition 2 *Assume that in a reference country, country j, the representative household maximizes (1) subject to (2), under the assumptions of Proposition 1. Assume now that the household of country j is confronted with the sequence of prices, per-capita profits and initial capital stock of country i. In a closed economy with no government, to a first order approximation, the difference in equivalent consumption between living in a generic country i versus country j can be written as:*

$$\log\widetilde{C}_t^{*,i} - \log C_t^{*,j} = \frac{\left(1-\beta^j\right)}{s_c^j}\left[E_t\sum_{s=0}^{\infty}(\beta^j)^s\left(\log\overline{PR}_{t+s}^i - \log PR_{t+s}^j\right) + \frac{1}{\beta^j}\left(\frac{P^{I,j}K^j}{P^{Y,j}Y^j}\right)\left(\log K_{t-1}^i - \log K_{t-1}^j\right)\right] \tag{13}$$

where the productivity terms are constructed using country j's shares in the following fashion:

$$\log\overline{PR}_{t+s}^i = \left(s_C^j\log C_{t+s}^i + s_I^j\log I_{t+s}^i\right) - s_L^j\log L_{t+s}^i - s_K^j\log K_{t+s-1}^i \tag{14}$$

and:

$$\log PR_{t+s}^j = \left(s_C^j\log C_{t+s}^j + s_I^j\log I_{t+s}^j\right) - s_L^j\log L_{t+s}^i - s_K^j\log K_{t+s-1}^i \tag{15}$$

Proof. See the Appendix. ■

Welfare differences across countries, therefore, are summarized by two components. The first component is related to the well-known log difference between TFP levels, which accounts empirically for most of the difference is per-capita income across countries (Hall and Jones (1999)). In the development accounting literature, this gap is interpreted as a measure of technological or institutional differences between countries. This interpretation, however, is valid under restrictive

assumptions on market behavior and technology (perfect competition, constant returns to scale, no externalities, etc.). We provide a different interpretation of cross-country differences in TFP that applies even when these assumptions do not hold, by showing that a slightly-modified version of the log-difference between TFP levels is essential for welfare comparisons across countries. The second component of the welfare difference reflects the difference in capital intensity between the two countries; *ceteris paribus*, a country with more capital per person can afford more consumption or higher leisure. The development accounting literature also uses capital intensity as the second variable explaining cross country differences in per-capita income.

Our result holds for any kind of technology and market structure, as long as a representative consumer exists, takes prices as given and is not constrained in the amount he can buy and sell at those prices. Notice however, that our measure of per-capita TFP is modified with respect to the traditional growth accounting measure in two ways. First, measuring welfare differences requires comparing not only current log differences in TFP but the present discounted value of future ones as well. Second, the distributional and expenditure shares used to compute the log differences in TFP between countries need to be calculated at their steady-state values in the reference country.[13]

As in the case of Proposition 1, we shall try to convey the economic reasoning for the result by considering the much simpler case of an economy with a zero steady-state growth rate ($g = 0$). Assume we confront the household from country j with the prices, profits and the initial per-capita capital stock of country i. If we expand the utility of a representative member of the household, denoted by $\widetilde{V}_t^i$, around the US steady-state we obtain:

$$\begin{aligned} \frac{(\widetilde{V}_t^i - V^j)}{\lambda^j p^{Y,j} Y^j} &= E_t \sum_{s=0}^{\infty} (\beta^j)^s [s_L^j (\log p_{t+s}^{L,i} - \log p^{L,j}) + s_K^j (\log\ p_{t+s}^{K,i} - \log\ p^{K,j}) \\ &+ s_\Pi^j (\log \pi_{t+s}^i - \log \pi^j) - s_C^j (\log p_{t+s}^{C,i} - \log p^{C,j})] \\ &+ \frac{1}{\beta^j} \left(\frac{P^{I,j} K^j}{P^{Y,j} Y^j} \right) \left(\log K_{t-1}^i - \log K^j \right) \end{aligned} \tag{16}$$

Now expand per-capita utility for the US around its own steady-state and subtract from (16). This yields:

$$\begin{aligned} \frac{(\widetilde{V}_t^i - V_t^j)}{\lambda^j p^{Y,j} Y^j} &= E_t \sum_{s=0}^{\infty} (\beta^j)^s [s_L^j (\log p_{t+s}^{L,i} - \log p_{t+s}^{L,j}) + s_K^j (\log\ p_{t+s}^{K,i} - \log\ p_{t+s}^{K,j}) \\ &+ s_\Pi^j (\log \pi_{t+s}^i - \log \pi_{t+s}^j) - s_C^j (\log p_{t+s}^{C,i} - \log p_{t+s}^{C,j})] \\ &+ \frac{1}{\beta^j} \left(\frac{P^{I,j} K^j}{P^{Y,j} Y^j} \right) \left(\log K_{t-1}^i - \log K_{t-1}^j \right) \end{aligned} \tag{17}$$

Differences in welfare across countries are, therefore, due to differences in their relative prices, per-capita profits and capital intensity.

Use (12) to express differences in utility across countries in term of log differences in equivalent

[13] It is standard in the development accounting literature to assume that all countries have the same capital and labor shares in income (often one-third and two-thirds), but to use country-specific shares in expenditure.

consumption on the left hand side of (17). Now linearize around country j's steady state two budget constraints: first, the budget constraint for the household from country j living in country i and, second, its budget constraint when it faces country j's conditions. Subtracting one from the other allows us to write the right hand side of (17) in terms of productivity differences and differences in the initial capital stock[14] This yields equations (13), (14), and (15) in Proposition 2.

Notice that in stating Proposition 2, we have not needed to assume that either the population growth rate n or the per-capita growth rate g is common across countries. We also have not assumed that any utility parameter (e.g., σ or β) is common across countries. This is because we are making the comparisons from the point of view of the representative individual in the reference country, who is faced with the exogenous (to the household) prices and lump-sum transfers of country i. This thought experiment uses the utility parameters of the reference country only, and does not require that exogenous variables change at the same long-run rate in the two countries. However, it may well be the case that our first-order approximation will give poor results unless the countries share a common long-run (and even medium-run) growth rate of per-capita variables.[15] This consideration suggests that it is more reasonable to focus on a subset of countries that are relatively homogenous, which is what we do by using core OECD countries in our empirical illustration.

3 Extensions

We now show that our method of using TFP to measure welfare can be extended to allow for the presence of taxes and government expenditure, multiple types of goods and labor, and an open economy setting. These extensions require modifications in the formulas given above for welfare comparisons over time and across countries, and we state the changes to the basic framework that are needed in each case; detailed derivations are given in the appendix. These results prove that the basic idea of using TFP to measure welfare holds in a variety of economic environments, but they also demonstrate the advantage of deriving the welfare measure from an explicit dynamic model of the household.

To save space, we focus on the measurement of welfare changes over time. Analogous results apply in all cases to the measurement of cross-country welfare differences, but to save space we leave the details for the Appendix.

[14] Expand around country j's steady state the budget constraint of the household from country j facing country i's relative prices, per capita profits and initial capital stock. Do the same for the budget constraint of a household from country i living in country i. Their optimal choice of consumption, capital accumulation and labor supply will differ, unless they share the same utility function, but, to a first order approximation, the algebraic sum of the terms in prices, profits and initial condition for the household from country j facing country i prices and endowments equals the algebraic sum of the terms representing consumption, labor supply and capital chosen by the individual from country i.

[15] Both introspection and the results of Kremer, Onatski and Stock (2001) suggest, however, that it is implausible to assume that countries will diverge perpetually in per-capita terms. Thus, even though we do not need to assume a common g, we would not view it as a restrictive assumption.

3.1 Taxes

Consider first an environment with distortionary and/or lump-sum taxes. Since the prices in the budget constraint (2) are those faced by the consumer, in the presence of taxes all prices should be interpreted as after-tax prices. At the same time, the variable that we have been calling "profits," Π_t, can be viewed as comprising any transfer of income that the consumer takes as exogenous. Thus, it can be interpreted to include lump-sum taxes or rebates. Finally one should think of B_t as including both government and private bonds (assumed to be perfect substitutes, purely for ease of notation).

More precisely, in order to modify (2) to allow for taxes, let τ_t^K be the tax rate on capital income, τ_t^R be the tax rate on revenues from bonds, τ_t^L be the tax rate on labor income, τ_t^C be the *ad valorem* tax on consumption goods, and τ_t^I be the corresponding tax on investment goods.[16] Also, let $P_t^{C\prime}$ and $P_t^{I\prime}$ respectively denote the pre-tax prices of consumption and capital goods, so that the tax-inclusive prices faced by the consumer are $P_t^{C\prime}\left(1+\tau_t^C\right)$ and $P_t^{I\prime}\left(1+\tau_t^I\right)$. We assume for the time being that the revenue so raised is distributed back to individuals using lump-sum transfers; we consider government expenditures in the next sub-section. The representative household's budget constraint now is

$$\begin{aligned} P_t^{I\prime}\left(1+\tau_t^I\right)K_tN_t + B_tN_t \;=\;& (1-\delta)P_t^{I\prime}\left(1+\tau_t^I\right)K_{t-1}N_{t-1} + \left(1+i_t^B\left(1-\tau_t^R\right)\right)B_{t-1}N_{t-1} \\ & + P_t^L\left(1-\tau_t^L\right)L_tN_t + P_t^K\left(1-\tau_t^K\right)K_{t-1}N_{t-1} + \Pi_tN_t - P_t^{C\prime}\left(1+\tau_t^C\right)C_tN_t \end{aligned} \tag{18}$$

Thus, differently than in the benchmark case, the exogenous variables in the household's maximization are not only the prices and the initial stocks of capital and bonds, but also the tax rates on labor and capital income, consumption and investment. However, as the appendix shows, the basic results (7) and (13) continue to hold, with the only modification that in defining the Solow productivity residual we need to take account of the fact that the national accounts measure factor payments as perceived by firms – that is, before income taxes – while nominal expenditure is measured using prices as perceived from the demand side, thus inclusive of indirect taxes (subsidies) on consumption and investment. Hence, letting $P_t^C \equiv P_t^{C\prime}\left(1+\tau_t^C\right)$ and $P_t^I \equiv P_t^{I\prime}\left(1+\tau_t^I\right)$ denote the tax-inclusive prices of consumption and investment goods, the expenditure shares s_C and s_I defined earlier are fully consistent with those obtained from national accounts data, but the factor shares s_L and s_K defined above refer to the gross income of labor and capital rather than their respective after-tax income. Thus, to be consistent with the data, in the presence of taxes the welfare residual needs to be redefined in terms of the shares of after-tax returns on labor and capital. Specifically, equation (5) can be re-written as:

[16] For simplicity, we are assuming no capital gains taxes and no expensing for depreciation. These could obviously be added at the cost of extra notation.

$$\Delta \log PR_{t+s} = \Delta \log Y_{t+s} - \left(1 - \tau^L\right) s_L \Delta \log L_{t+s} - \left(1 - \tau^K\right) s_K \Delta \log K_{t+s-1} \tag{19}$$

and an analogous modification applies to (14) and (15) (see the appendix). τ^L and τ^K are the steady-state values of τ_t^L and τ_t^K With these modifications, our results generalize to a setting with distortionary time-varying taxes on consumption and investment goods and on the household income coming from labor, capital or financial assets.

3.2 Government Expenditure

With some minor modification, our framework can be likewise extended to allow for the provision of public goods and services. We illustrate this under the assumption that government activity is financed with lump-sum taxes. Using the results from the previous subsection, it is straightforward to extend the argument to the case of distortionary taxes.

Assume that government spending takes the form of public consumption valued by consumers. We rewrite instantaneous utility as

$$U(C_{t+s}, C_{G,t+s}, L_{t+s}) = \frac{1}{1-\sigma} C(C_{t+s}; C_{G,t+s})^{1-\sigma} \nu(\overline{L} - L_{t+s}) \tag{20}$$

where C_G denotes per-capita public consumption and $C(.)$ is homogenous of degree one in its arguments. Total GDP now includes public consumption, that is, $P_t^Y Y_t = P_t^C C_t + P_t^G C_{G_t} + P_t^I I_t$, where P^G is the public consumption deflator.

In this setting, our earlier results need to be modified to take account of the fact that public consumption may not be set by the government at the level that consumers would choose. Intuitively, in such circumstances the value that consumers attach to public consumption may not coincide with its observed value as included in GDP, and therefore in the productivity residual as conventionally defined.

Formally, let $s_{cGt} = \frac{P_t^G C_{G_t}}{P_t^Y Y_t}$ denote the GDP share of public consumption, and let $s^*_{cG_t}$ denote the share that would obtain if public consumption were valued according to its marginal contribution to the utility of the representative household, rather than using its deflator P_t^G.[17] The welfare-relevant modified Solow residual (5) now is

$$\Delta \log PR_{t+s} = \Delta \log Y_{t+s} - s_L \Delta \log L_{t+s} - s_K \Delta \log K_{t+s-1} + \left(s^*_{cG} - s_{cG}\right) \Delta \log C_{G,t+s} \tag{21}$$

and an analogous modification applies to (14) and (15). Hence in the presence of public consumption the Solow residual needs to be adjusted up or down depending on whether public consumption is under- or over-provided (i.e., $s^*_{cG} > s_{cG}$ or $s^*_{cG} < s_{cG}$ respectively). If the government

[17]It is easy to verify that $s^*_{cG_t} = \frac{U_{CGt} P_t^C C_{G_t}}{U_{C_t} P_t^Y Y_t}$.

sets public consumption exactly at the level the utility-maximizing household would have chosen if confronted with a price of P_t^G, then $s_{c_G}^* = s_{c_G}$ and no correction is necessary. In turn, in the standard neoclassical case in which public consumption is pure waste $s_{c_G}^* = 0$, the welfare residual should be computed on the basis of private final demand – i.e., GDP minus government purchases. With the residual redefined in this way, the growth rate of equivalent consumption now is[18]

$$\Delta \log (C_t)^* = \frac{(1-\beta)}{(s_C + s_{C_G}^*)} \left[E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} + \sum_{s=0}^{\infty} \beta^s \Delta E_t \log PR_{t+s} + \frac{1}{\beta} \left(\frac{P^I K}{P^Y Y} \right) \Delta \log K_{t-1} \right] \tag{22}$$

3.3 Multiple Types of Labor, Consumption and Investment Goods

The extension to the case of multiple types of labor, consumption and investment goods is immediate. For simplicity, we can assume that each individual is endowed with the ability to provide different types of labor services, $L_{h,t}$ and that the instantaneous utility function can be written as:

$$U(C_{1,t+s}, .., C_{Z,t+s}, L_{1,t+s}, ..., L_{H_L,t+s}) = \frac{1}{1-\sigma} C(C_{1,t+s}, .., C_{Z,t+s})^{1-\sigma} \nu \left[\overline{L} - L(L_{1,t+s}, ..., L_{H_L,t+s}) \right] \tag{23}$$

where $L(.)$ and $C(.)$ are homogenous functions of degree one, H_L is the number of types of labor and Z is the number of consumption goods. Denote by $P_t^{C_h}$ the price of a unit of $C_{h,t}$,and by $P_t^{L_h}$ the payment to a unit of $L_{h,t}$.[19] Similarly, assume that consumers can purchase H_I different types of investment goods $I_{h,t}$ at prices $P_t^{I_h}$, and combine them into capital according to a constant-returns aggregate investment index. Thus, investment (in per-capita terms) can be expressed $I_t = I(I_{1,t}, ..., I_{H_I,t})$, and the trajectory of the capital stock is still described by equation (3). Further, we can define (exact) deflators for labor income, consumption and investment P_{t+s}^L, P_{t+s}^C and P_{t+s}^I, each of which is a linear homogenous function of the prices of the underlying individual types of labor and goods, respectively, such that $P_{t+s}^L L_{t+s} = \sum_{h=1}^{H_L} P_{t+s}^{L_h} L_{h,t+s}$; $P_{t+s}^C C_{t+s} = \sum_{h=1}^{Z} P_{t+s}^{C_h} C_{h,t+s}$ and $P_{t+s}^I I_{t+s} = \sum_{h=1}^{H_I} P_{t+s}^{I_h} I_{h,t+s}$.[20]

In this framework, our earlier results continue to hold without modification: the applicable expression for the modified Solow residual still is (5), and welfare changes over time and differences across countries continue to be characterized by (7) and (13) respectively. The only new feature is that the GDP shares of labor, consumption and investment can also be expressed as the sums of

[18] Government purchases might also yield productive services to private agents. For example, the government could provide education or health services, or public infrastructure, which – aside from being directly valued by consumers – may raise private-sector productivity. In such case, the results in the text remain valid, but it is important to note that the contribution of public expenditure to welfare would not be fully captured by the last term in the modified Solow residual as written in the text. To this term we would need to add a measure of the productivity of public services, which is implicitly included in the other terms in the expression.

[19] We assume that the nature of the utility function is such that positive quantities of all types of labor are supplied.

[20] The existence of these perfect price indices under the assumptions made in the text was established by the classic literature on two-stage consumption budgeting; see Lloyd (1977). The extension to investment is discussed by Servén (1995).

the shares of their respective disaggregated components evaluated at the steady state; e.g., $s_{L_t} = \frac{P_t^L L_t}{P_t^Y Y_t} = \sum_{h=1}^{H_L} s_{L_{h,t}}$, where $s_{L_h,t} = \frac{P_t^{L_h} L_{h,t}}{P_t^Y Y_t}$. Moreover, $\Delta \log L_t$, $\Delta \log C_t$, and $\Delta \log I_t$ are Divisia indices (with fixed weights) of individual types of labor, consumption and investment.

3.4 Open Economy

Our results apply also to an open economy if we just replace GDP with domestic absorption in the definition of the Solow residual, and thus rewrite (5) as:

$$\Delta \log PR_{t+s} = \Delta \log A_{t+s} - s_L \Delta \log L_{t+s} - s_K \Delta \log K_{t+s-1} \tag{24}$$

where $\Delta \log A_t = s_C \Delta \log C_t + s_I \Delta \log I_t$ is a Divisia index of domestic absorption growth (in real per-capita terms), and s_L, s_K, s_C and s_I are shares out of domestic absorption. $\Delta \log C_t$ and $\Delta \log I_t$ are, in turn, Divisia indices of domestically produced and imported goods aggregated with fixed weights, as discussed in the previous subsection. In addition, the steady-state capital-output ratio in (7) and ensuing expressions in the preceding section should also be replaced with the steady-state capital-absorption ratio. It is true that now the initial stock of net foreign bonds, and the return on those bonds, would appear – along with the initial capital stock, profits, and prices – as a determinant of utility in equation (8). However they would also appear in the budget constraint (10) and would cancel out in the primal version of the productivity residual, provided the latter is defined in terms of absorption. As a consequence, all the results we have stated in terms of the primal productivity residual continue to hold[21].

Alternatively, one may want to use a standard measure of output, real GDP, defined as consumption, plus investment, plus net exports. Then the welfare-relevant residual can be written as the sum of a conventionally-defined productivity residual plus additional components that capture terms of trade and capital gains effects. Moreover, the initial conditions then should include the initial value of the net foreign asset stock. To show this, assume that the domestic economy buys imports $IM_{t+s}N_{t+s}$ at a price P_{t+s}^{IM} and sells domestic goods abroad $EX_{t+s}N_{t+s}$ at a a price P_{t+s}^{X}. The current account balance can be written:

$$B_t N_t - B_{t-1} N_{t-1} = i_t^B B_{t-1} N_{t-1} + P_t^{EX} EX_t N_t - P_t^{IM} IM_t N_t \tag{25}$$

where B_t is taken to denote the per-capita foreign asset stock. The applicable Divisia index of per capita GDP growth now is $\Delta \log Y_t = s_C \Delta \log C_t + s_I \Delta \log I_t + s_X \Delta \log EX_t - s_M \Delta \log IM_t$, where s_C, s_I, s_X and s_M are respectively the steady-state shares of consumption, investment, exports and imports out of total value added.

[21] We could also allow for bond financing of government expenditure. The existence of government bonds does not change our results when they are expressed in terms of the primal version of the productivity residual.

Using these definitions, the appendix shows that welfare changes can be related to a productivity residual corrected for terms of trade changes and changes in the rate of return on foreign assets:

$$\begin{aligned}\Delta \log PRTT_{t+s} \quad = \quad & \Delta \log Y_{t+s} - s_L \Delta \log L_{t+s} - s_K \Delta \log K_{t+s-1} \\ & + s_X \Delta \log p^{EX}_{t+s} - s_M \Delta \log p^{IM}_{t+s} + \left(\frac{Br}{P^Y Y}\right) \Delta \log r_{t+s}\end{aligned} \tag{26}$$

where s_L and s_K are also shares out of total value added, r is the real rate of return on foreign assets, and $\left(\frac{Br}{P^Y Y}\right)$ is the steady-state ratio of foreign asset income to GDP. Changes in welfare can be summarized by an expression similar to (7), but based on (26) rather than the conventional Solow residual:

$$\begin{aligned}\Delta \log C_t^* \quad = \quad & \frac{(1-\beta)}{s_c} \left[E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PRTT_{t+s} + \sum_{s=0}^{\infty} \beta^s \Delta E_t \log PRTT_{t+s} \right. \\ & \left. + \frac{1}{\beta} \left(\frac{P^I K}{P^Y Y}\right) \Delta \log K_{t-1} + \frac{1}{\beta} \left(\frac{B}{P^Y Y}\right) \Delta \log \frac{B_{t-1}}{P^I_{t-1}} \right]\end{aligned} \tag{27}$$

Conceptually, it makes very good sense that all these extra terms come into play when taking the GDP route to the measurement of welfare in the open economy. The effects of an improvement in the terms of trade, as captured by $s_X \Delta \log p^{EX}_t - s_M \Delta \log p^{IM}_t$ in (26), are analogous to those of an increase in TFP - both give the consumer higher consumption for the same input of capital and labor (and therefore higher welfare); see Kohli (2004) for a static version of this result. In turn, the term in $\Delta \log r_{t+s}$ captures present and expected future changes in the rate of return on foreign assets, including capital gains and losses on net foreign assets due either to exchange rate movements or to changes in the foreign currency prices of the assets. Finally, the initial conditions now include not only the (domestic) capital stock, but also the net stock of foreign assets.

Measuring these extra terms empirically poses major challenges, however. One needs reliable measures of changes in foreign asset holdings for a large sample of countries. Asset returns would have to be measured in risk-adjusted terms to make them comparable across countries. In addition, forecasts of future asset returns and the terms of trade would be required as well. In contrast, all these problems disappear if the measurement of welfare is based on real absorption rather than GDP as the measure of output, in which case the same terms that summarize welfare in the closed economy suffice to measure it in the open economy. The implication is that we can measure welfare empirically in ways that are invariant to the degree of openness of the economy.

3.5 Multiple Wages and Labor Market Rationing

So far we have assumed that the household is a price-taker in goods and factor markets, and that it faces no constraints other than the intertemporal budget constraint. We have exploited the insight

that under these conditions relative prices measure the representative consumer's marginal rate of substitution between goods, even when relative prices do not measure the economy's marginal rate of transformation. We now ask whether our conclusions need to be modified in environments where the household does not behave as a price taker. We present two examples, and then draw some tentative conclusions about the robustness of our previous results.

Our examples focus on the labor market. It seems reasonable to assume that consumers are price-takers in capital markets; most individuals take rates of return on assets as exogenously given. The assumption is still tenable when it comes to the purchase of goods, although some transaction prices may be subject to bargaining. The price-taking assumption seems most questionable when it comes to the labor market, and indeed several literatures (on labor search, union wage setting, and efficiency wages, to name three) begin by assuming that households are not price takers in the labor market. Thus, we study two examples. One is in the spirit of the dual labor markets literature, where wages are above their market-clearing level in some sectors but not in other. We do not model why wages are higher in the primary sector, but this can be due to the presence of unions or government mandates in formal but not in informal employment, or efficiency wage considerations in some sectors but not in others. Wages in the secondary market are set competitively. The second example is in the spirit of labor market search, and has households face a whole distribution of wages. In both cases households would prefer to supply all their labor to the sector or firm that pays the highest wage, but are unable to do so. In this sense, both examples feature a type of labor market rationing. (In both cases the different wages are paid to identical workers, and are not due to differences in human capital characteristics.)

First, consider the case in which the household can supply labor in two labor markets. The primary market pays a high wage $\overline{P_t^L}$ and the secondary market pays a lower wage $\underline{P_t^L} < \overline{P_t^L}$. Although the worker prefers to work only in the primary sector, the desirable jobs are rationed; he cannot supply more than $\widetilde{L}$ hours in the high-wage job in each period. The representative household faces the following budget constraint:

$$P_t^I K_t N_t + B_t N_t = (1-\delta) P_t^I K_{t-1} N_{t-1} + \left(1 + i_t^B\right) B_{t-1} N_{t-1} + N_t \overline{P_t^L} \widetilde{L} + N_t \underline{P_t^L} (L_t - \widetilde{L}) + P_t^K K_{t-1} N_{t-1} + \Pi_t N_t - P_t^C N_t \tag{28}$$

Assuming that the labor rationing constraint is binding, the logic of our previous derivations remains valid, but now we need to re-define the labor share in terms of the lower wage paid in the secondary market. For instance the modified productivity growth residual for the closed economy is now:

$$\Delta \log PR_t = \log Y_t - \Delta \underline{s_L} \log L_{h,t} - s_K \Delta \log K_{h,t-1} \tag{29}$$

where the distributional share of labor $\underline{s_L} \equiv \frac{\underline{P^L} L}{P^Y Y}$ is computed using the *low* wage, paid in the dual labor market, rather than the average wage. The intuition for this result comes from the fact that the marginal wage for the household is $\underline{P_t^L}$ while $N_t(\overline{P_t^L} - \underline{P_t^L})\widetilde{L}$ can be considered as a lump-sum transfer and can be treated exactly like the profit term in the budget constraint. (Thus, we can

also allow for arbitrary variations over time in the primary wage $\overline{P_t^L}$ or the rationed number of hours $\widetilde{L}$ without changing our derivations.)

This example shows that in some cases our methods need to be modified if the household is no longer a price-taker. However, in this instance the modification is not too difficult—one can simply decrease the labor share by the ratio of the average wage to the competitive wage. Furthermore, this example shows that imperfect competition in factor markets can introduce an additional gap between the welfare residual and the standard Solow residual that is like a tax wedge, making our modifications to standard TFP even more important if one wants to use TFP data to capture welfare. As is the case with taxes, welfare rises with increases in output holding inputs constant, even if there is no change in actual technology.

Note that we would get a qualitatively similar result if, instead of labor market rationing, we assumed that the household has monopoly power over the supply of labor, as in many New Keynesian DSGE models. As in the example above, we would need to construct the true labor share by using the household's marginal disutility of work, which would be less than the real wage. In this environment, we would obtain the welfare-relevant labor share by dividing the observed labor share by an assumed value for the average markup of the wage over the household's marginal rate of substitution between consumption and leisure.

The second example shows that there are situations where our previous results in sections 2 and 3 are exactly right and need no modification, even with multiple wages and labor market rationing. Consider a household that comprises a continuum of individuals with mass N_t. Suppose that each individual can either not work, or work and supply a fixed number of hours $\widehat{L}$. In this environment, the household can make all its members better off by introducing lotteries that convexify their choice sets. Suppose that the household can choose the probability q_t for an individual to work. The representative household maximizes intertemporal utility:

$$W_t = \sum_{s=0}^{\infty} \frac{1}{(1+\rho)^s} \frac{N_{t+s}}{H} \left[q_t U(C_t^0; \overline{L} - \widehat{L}) + (1-q_t) U(C_t^1; \overline{L}) \right] \tag{30}$$

where $q_t U(C_t^0; \overline{L} - \widehat{L}) + (1-q_t) U(C_t^1; \overline{L})$ denotes expected utility prior to the lottery draw. C_t^0 and C_t^1 denote respectively per-capita consumption of the employed and unemployed individuals, while average per-capita consumption, C_t is given by:

$$C_t = q_t C_t^0 + (1-q_t) C_t^1 \tag{31}$$

Assume that the individuals that work face an uncertain wage P_t^L, which is observed only after labor supply decisions have been made. More specifically, individual wages in period t are iid draws from a distribution with mean $E_t P_t^L$. Notice that, by the law of large numbers, the household does not face any uncertainty regarding its total wage income. Thus, the budget constraint for the household becomes:

$$P_t^I K_t N_t + B_t N_t = (1-\delta) P_t^I K_{t-1} c + \left(1 + i_t^B\right) B_{t-1} N_{t-1} + q_t N_t E_t P_t^L \widehat{L} + P_t^K K_{t-1} N_{t-1} + \Pi_t N_t - P_t^C \left(q_t C_t^0 + (1-q_t) C_t^1 \right. \tag{32}$$

Following Rogerson and Wright (1988) and King and Rebelo (1999),[22] we can rewrite the per-period utility function as:

$$U(C_t; L_t) = \frac{1}{1-\sigma} {C_t}^{1-\sigma} \nu^*(L_t) \tag{33}$$

where $L_t = q_t \widehat{L}$ denotes the average number of hours worked and:

$$\nu^*(L_t) = \left(\frac{L_t}{\widehat{L}} \nu(\overline{L} - \widehat{L})^{\frac{1}{\sigma}} + (1 - \frac{L_t}{\widehat{L}}) \nu(\overline{L})^{\frac{1}{\sigma}} \right)^{\sigma} \tag{34}$$

In summary, the maximization problem faced by the household is exactly the same as the one described in section 2, even if identical workers are paid different wages [23]. All the results we have derived previously also apply in this new setting. The second example leads to a different result from the first for two reasons. First, it is an environment with job search rather than job queuing. Second, the number of hours supplied by each worker is fixed. Under these two assumptions, the labor supply decision is made *ex ante* and not *ex post*.

From these two examples, it is clear that dropping the assumption that all consumers face the same price for each good or service can—but need not—change the precise nature of the proxies we develop for welfare. Even in the case where the measure changed, however, our conclusion that welfare can be summarized by a forward-looking TFP measure and capital intensity remained robust. While the exact nature of the proxy will necessarily be model-dependent, we believe that our basic insight applies under fairly general conditions.

3.6 Summing up

We can now go back to the two propositions stated earlier. They were formulated for the special case of a closed economy with no government. In light of the discussion in this section, we can now restate them in a generalized form for an open economy with multiple goods and a government sector, which is more amenable for empirical implementation. We will continue to assume price taking behavior in all markets.

Proposition 1' *Assume an open economy in which the government engages in public con-*

[22] In obtaining this result we use the fact that the marginal utility of consumption of the individuals in the household needs to be equalized at the optimum. This implies: $c_t^0 = c_t^1 \left(\frac{\nu(\overline{L}-\widehat{L})}{\nu(\overline{L})}\right)^{\frac{1}{\sigma}}$.

[23] King and Rebelo (1999) show that in this framework the representative agent has an infinite Frisch labor supply elasticity. This result follows from the assumption that all agents in the household have the same disutility of labor. Mulligan (2001) shows that even when all labor is supplied on the extensive margin, one can obtain any desired Frisch elasticity of labor supply for the representative agent by allowing individual agents to have different disutilities of labor. In a more elaborate example, we could use Mulligan's result to show that the only restrictions on the preferences of the representative agent are those that we assume in Section 2.

sumption, and levies taxes on labor and capital income at rates τ^L_{t+s} and τ^K_{t+s}, as well as taxes on consumption and investment expenditure at rates τ^C_{t+s} and τ^I_{t+s}. Assume also that the representative household maximizes intertemporal utility taking prices, profits, interest rates, tax rates and public consumption as exogenously given. Lastly, assume that population grows at a constant rate n, and the wage and all per-capita quantities other than labor hours grow at rate g in the steady-state. To a first-order approximation, the growth rate of equivalent consumption can be written as:

$$\Delta \log (C_t)^* = \frac{(1-\beta)}{(s_C + s^*_{C_G})} \left[E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} + \sum_{s=0}^{\infty} \beta^s \Delta E_t \log PR_{t+s} + \frac{1}{\beta} \left(\frac{P^I K}{P^A A} \right) \Delta \log K_{t-1} \right] \tag{35}$$

where productivity growth is defined as

$$\Delta \log PR_{t+s} = s_C \Delta \log C_{t+s} + s_I \Delta \log I_{t+s} + s^*_{C_G} \Delta \log C_{G,t+s} - \left(1-\tau^L\right) s_L \Delta \log L_{t+s} - \left(1-\tau^K\right) s_K \Delta \log K_{t+s-1} \tag{36}$$

All shares are defined relative to total absorption, A_t. $\log C_t$, and $\log I_t$ are share weighted aggregates (with fixed weights) of individual types of domestically produced and imported consumption and investment goods. Shares and tax rates are evaluated at their steady-state values, and $s^*_{C_G}$ denotes the steady-state share of public consumption in total absorption that would obtain if public consumption were valued according to its marginal contribution to the utility of the representative household.

Proof. See the Appendix. ■

By stating (35), as well as all the expenditure and income shares entering the productivity residual, in terms of absorption, the proposition applies to both open and closed economies. Further, as explained earlier, the value of $s^*_{C_G}$ depends on the assumptions made about government consumption: if the latter is set at the level the representative household would have chosen if she were facing the price P^G, then $s^*_{C_G} = s_{C_G} \equiv \frac{P^G C_G}{P^C C + P^I I + P^G C_G}$. Alternatively, if government consumption is pure waste (i.e., if it does not enter the consumption aggregate C in (20)), $s^*_{C_G} = 0$. This implies that the proposition can encompass a variety of cases with respect to taxation and government spending: 1) wasteful government spending with lump sum taxes (in which case distortionary taxes are set to zero in the productivity equation); 2) optimal government spending with lump sum taxes; 3) wasteful government spending with distortionary taxes; 4) optimal government spending with distortionary taxes.

Our main result regarding welfare differences across countries can be restated in a similar way:

Proposition 2' *Assume that in a reference country, country j, the representative household maximizes intertemporal utility under the assumptions of Proposition 1'. Assume now that the household of country j, is confronted with the sequence of prices, per-capita profits, public consumption and and endowment of country i. In an open economy with distortionary taxation and*

government spending, the difference in equivalent consumption between living in a generic country i versus country j can be written, to a first order approximation, as:

$$\ln \widetilde{C}_t^{*,i} - \ln C_t^{*,j} = \frac{(1-\beta^j)}{(s_C^j + s_{C_G}^{*j})} \left[E_t \sum_{s=0}^{\infty} (\beta^j)^s \left(\log \overline{PR}_{t+s}^i - \log PR_{t+s}^j \right) + \frac{1}{\beta^j} \left(\frac{P^{I,j} K^j}{P^{A,j} A^j} \right) \left(\log K_{t-1}^i - \log K_{t-1}^j \right) \right] \tag{37}$$

where A^j denotes absorption and $s_{C_G}^{*j}$ the steady-state share of public consumption in total absorption that would obtain if public consumption were valued according to its marginal contribution to the utility of the representative household. The two productivity terms are constructed with all shares (in terms of absorption) and tax rates evaluated at the reference country's steady-state values:

$$\log PR_t^j = s_C^j \log C_t^j + s_I^j \log I_t^j + s_{C_G}^{*j} \log C_{G,t}^j - \left(1 - \tau^{L,j}\right) s_L^j \log L_t^j - \left(1 - \tau^{K,j}\right) s_K^j \log K_{t-1}^j \tag{38}$$

$$\log \overline{PR}_t^i = s_C^j \log C_t^i + s_I^j \log I_t^i + s_{C_G}^{*j} \log C_{G,t}^i - \left(1 - \tau^{L,j}\right) s_L^j \log L_t^i - \left(1 - \tau^{K,j}\right) s_K^j \log K_{t-1}^i \tag{39}$$

where $\log C$ and $\log I$ are share weighted aggregates (with the fixed weights of the reference country) of individual types of domestically produced and imported consumption and investment goods.

Proof. The proof follows from the proof of Proposition 2, together with the extensions contained in Proposition 1'. ■

Proposition 2' shows that our method for comparing welfare across countries applies in a much more general setting than the one we used previously. We can compare two economies with any degree of openness to trade or capital flows, and with differing levels of distortionary taxation or government expenditure. The derivation shows a result that would be hard to intuit ex ante, which is that to a first-order approximation only the tax rates of the reference country enter the welfare comparison[24]. This asymmetry implies that welfare rankings may depend on the choice of reference country. In our empirical application below we take the US as our reference country, but check the robustness of the results by using France instead[25].

4 Empirical results

4.1 Data and Measurement

In order to discuss how our index of welfare changes over time within a country and how it compares across countries, we use yearly data on consumption, investment, capital and labor services for the years 1985-2005 and for seven countries: the US, the UK, Japan, Canada, France, Italy and Spain.

[24] Of course, the tax rates of the comparison country will generally change output and input levels in that country through general-equilibrium effects, which will influence the welfare gap between the two countries. However, the tax rates of the comparison country do not enter the formula directly.

[25] We conjecture that the asymmetry may be eliminated by moving to second-order approximations, where instead of using the tax-adjusted shares of the reference country only, one might take an arithmetic average of the shares of the reference and comparison countries. We are investigating this possibility in current research.

We are unable to include Germany in the sample, since data for unified Germany are available only since 1995 in EU-KLEMS. We use two different data sets to compare welfare within a country and across countries.

To analyze welfare changes over time within a country, we combine data coming from the OECD Statistical Database and the EU-KLEMS dataset[26]. Our index of value added is constructed from the OECD dataset as the weighted growth of household final consumption, gross capital formation and government consumption (where appropriate) at constant national prices, using as weights their respective shares of value added. According to our theory, these shares should be kept constant at their steady-state level, but in practice we use shares averaged across the twenty years in our sample.

In constructing the growth rate of the "modified" productivity residual, we subtract from the log-changes in value added, the log changes in the capital and labor stocks used to produce it, weighted by their average respective shares out of total compensation. Data on aggregate production inputs are provided by EU-KLEMS. Log-changes in capital are constructed using the estimated capital stock constructed by applying the perpetual inventory method on investment data. In our benchmark specification, log-changes in the labor stock are approximated by log-changes in the amount of hours worked by persons engaged. Alternatively, we use a labor service index which is computed as a translog function of types of workers engaged (classified by skill, gender, age and sex), where weights are given by the average share of each type of worker in the value of total labor compensation. We assume that economic profits are zero in the steady-state so that we can recover the gross (tax unadjusted) share of capital as one minus the labor share.

In order to compare welfare across countries, we combine data coming from the Penn World Tables and the EU-KLEMS dataset. More specifically, our basic measure of value added is constructed from the Penn World Tables as the weighted average of PPP converted log-private consumption, log-gross investment and log-government consumption, using as weights their respective shares of value added in the reference country; as in the within case, we use shares that are averaged across the twenty years in our sample.

To construct the modified log-productivity residual in each country, we subtract from this measure of value added the amount of capital and the amount of labor used to produce it, weighted by their respective share of compensation in the reference country, also in this case kept constant at their average value. The stock of capital in the economy is constructed using the perpetual inventory method on the PPP converted investment time series from the Penn World Tables. The stock of labor is computed in two different ways using the EU-KLEMS dataset. In our benchmark specification, the amount of labor services used in the economy is approximated by the hours worked. In the alternative specification, it is computed by aggregating over different types of persons engaged using a translog function, where weights are given by the shares of compensation to each type of labor out of total compensation to labor and are kept constant at their average value in the reference country.

[26] The EU-KLEMS data are extensively documented by O'Mahony and Timmer (2009).

Finally, to correct our welfare calculations for the presence of distortionary taxation, we use data on average tax rates on capital and labor provided by Boscá et al. (2005). The tax rates are computed by combining realized tax revenues, from the OECD Revenue Statistics, with estimates of the associated tax bases derived from the OECD National Accounts. These data update the tax rates constructed by Mendoza et al (1994) and introduce some methodological improvements in their calculation, most of which are described in Carey and Tchilinguirian (2000). In essence, they involve some adjustments to the definition of the various tax bases.

4.2 Within Results

Since the change in welfare over time depends on the expected present discounted value of TFP growth and its revision, as shown by equation (35), we need to construct forecasts of future TFP. In order to keep our empirical illustration simple and uniform across countries, we estimate univariate time-series models using annual data for the seven countries in our data set. The extension to a multivariate forecasting framework is something worth exploring in future work. Our sample period runs from 1985 to 2005 for all countries except Canada, where the EU-KLEMS data end in 2004.

We use the various aggregate TFP measures suggested by our theory (in log levels), and estimate simple AR processes for each country. The persistence of TFP growth is a key statistic, since it shows how the entire summation of expected productivity residuals changes as a function of the innovation in the log level of TFP. We report the persistence of TFP using simple, reduced-form forecasting equations for two different definitions of TFP, which we will use as benchmarks throughout.

The first concept is TFP in the case where we assume that government purchases are wasteful, and taxes are lump-sum. For this case, as discussed above, we construct output by aggregating consumption and investment only, but using shares that sum to $(1 - s_{c_G})$, and we do not correct the capital and labor shares for the effects of distortionary taxes. In this case, the capital and labor shares sum to one. The second case is the one where we assume government spending is optimally chosen, but needs to be financed with distortionary taxes. In this case, the output concept is the share-weighted sum (in logs) of consumption, investment and government purchases, and the capital and labor shares are corrected for both income taxes and indirect taxes (both of which reduce the after-tax shares). Note that in both cases the output concept measures absorption rather than GDP (unless the economy is closed, in which case the two concepts coincide). Thus, following our discussion in Section 3.4, both concepts (and indeed all the TFP measures that we use in this section) are appropriate for measuring welfare in both closed and open economies. In both cases, we assume that pure economic profits are zero in the steady-state.

For all countries, the log level of TFP is well described by either an AR(1) or AR(2) stationary process around a linear trend. In Table (1) we report the estimation results obtained using the two definitions of TFP stated above, together with the Lagrange Multiplier test for residual first order serial correlation (shown in the last line of each panel in the table), confirming that we cannot reject the null of no serial correlation for the preferred specification for each country. For all countries,

the order of the estimated AR process is invariant to the TFP measure used. In all cases, we can comfortably reject the null of a unit root in the log TFP process (after allowing for a time trend). We use the estimated AR processes to form expectations of future levels or differences of TFP, which are required to construct our welfare indexes.

We use equation (35), to express the average welfare change per year in each country in terms of changes in equivalent consumption. Given the time-series processes for TFP in each country, we can readily construct the first two terms in equation (35), the present value of expected TFP growth, and the change in expectations of that quantity. The third term, which depends on the change in the capital stock can also be constructed using data from EU-KLEMS. We assume that the composite discount rate, β, is common across countries and we set it equal to 0.95.[27] For the expenditure and distributional shares, we use their country specific averages over the sample period.

The results are in Table 2. We see that assumptions about fiscal policy affect the results in significant ways. We first illustrate our methods by discussing the results for the US, which are given in the last row. We then broaden our discussion to draw more general lessons from the full set of countries.

In the first column of Table 2, we construct the output data and the capital and labor shares under the assumption that government expenditure is wasteful and taxes are lump-sum. In this column, "utility-relevant output" comprises just consumption and investment, aggregated using weights that sum to less than one.[28] In this case, the average annual growth rate of welfare in the US is equivalent to a permanent annual increase in consumption of about 2.5 percent. Recall from Section 3.4 that this result applies whether we think of the US as an open or a closed economy. The same is true for all the other results in Table 2–all apply to open as well as closed economies.

Now we study the case of optimal government spending, still under the assumption that taxes are not distortionary. Thus, at the margin the consumer is indifferent between an additional unit of private consumption and an additional unit of government expenditures. In this case, output consists of consumption, investment and government purchases, aggregated using nominal expenditure shares that sum to one. In a closed economy this concept of TFP corresponds to the standard Solow residual. Note that in all cases our output concepts correspond to different measures of absorption; this is why they are relevant for both closed and open economies.

Welfare growth for the US is only slightly higher when we assume that expenditures are optimal: 2.6 versus 2.5 percent for the lump-sum tax cases. (We will see that this result is not universal within our sample of countries.) On the whole, the differing assumptions about the value of government expenditure do not change the calculated US growth rate of welfare significantly. Note, however, that this result does *not* mean that the US consumer is indifferent between wasteful and optimal government spending. Steady-state welfare is surely much lower in the case where the

[27] We construct our measure of β following the method of Cooley and Prescott (1995), who find $\beta = 0.947$.

[28] The weight on consumption is the nominal value of consumption divided by nominal expenditures on consumption, investment and government purchases. The weight on investment is the nominal value of investment, divided by the same denominator.

government wastes 20 percent of GDP. However, our results show that the difference in welfare in the two cases is almost entirely a *level* difference rather than a *growth rate* difference.

We repeat our welfare calculations under the assumption that the government raises revenue via distortionary taxes. The results are in columns 3 and 4 of Table 2. As shown above, if taxes are distortionary we need to construct the factor shares in the Solow residual using the after-tax wage and capital rental rate perceived by the household, implying that the shares will sum to less than one. We construct the new shares using the tax rates described in the previous section. The quantitative effect of this change is significant. In both of the cases we consider (wasteful spending and optimal spending), per-capita welfare growth expressed in terms of consumption growth rates is *higher* by nearly half a percentage point per year. Intuitively, if taxes are distortionary then steady-state output is too low; thus, any increase in output, even with unchanged technology, is a welfare improvement. It is quantitatively important to allow for the fact that taxes are distortionary and not lump-sum. For the US, it matters more for the growth rate of welfare than whether we assume that government spending is wasteful or optimal. We take as our benchmark the case shown in the last column, where spending is optimally chosen (from the point of view of the household) and taxes are distortionary. In this case, average US welfare growth is equivalent to a growth rate of per-capita consumption of 3 percent per year.

Assumptions about fiscal policy naturally matter more in countries with a high rate of growth of government purchases per-capita and with a high growth rate of factor inputs. For example, both facts are true for Spain over our sample period. The growth rate of welfare in Spain nearly doubles from the first column, where its 2.1 percent annual welfare growth rate is literally middling, to the last, where its 4 percent growth rate is the highest among all the countries in our sample. Assumptions about fiscal policy also matter significantly for Canada and Japan, and change the welfare growth rates of these countries by a full percentage point or more. In percentage terms, the change is particularly dramatic for Canada. Under three of the four scenarios, the UK leads our sample of countries in welfare growth rates; in the last case, it is basically tied with Spain.

Finally, we show the full time series of the welfare indexes for each country graphically, for our two benchmark cases of wasteful spending with lump-sum taxes and optimal spending with distortionary taxes. In Figures 1 and 2 we report the evolution over time of our welfare indexes for each country, in log deviations from their values in 1985. In Figure 1, the UK is the clear growth leader, with France and the US nearly tied in a second group, and Canada trailing badly. In Figure 2, by contrast, there are three clear groups: the UK and Spain lead, by a considerable margin; the US, France, and Japan comprise the middle group; Italy and Canada have the lowest welfare growth rates. Two countries show significant declines in growth rates, both starting in the early 1990s. The first is Japan, which in the first few years of our sample grew in line with the leading economies, Spain and the UK, and then slowly drifted down in growth rate to end the sample in the middle group, with France and the US. Similarly, Italy used to grow at the pace of the middle group, but then experienced a slowdown which, by the end of the sample, caused it to leave the middle group and form a low-growth group with Canada. Thus, our results are

consistent with the general impression that Italy and Japan experienced considerable declines in economic performance over the last two decades relative to the performance in the earlier postwar period.

In Table 3, we investigate which of the two components of welfare–TFP growth or capital accumulation–contributed more to the growth rate of welfare in our sample of countries. For the purpose of this decomposition, we treat the expectation-revision term as a contribution to TFP. In order to keep the table uncluttered, we drop the case where government spending is optimal and taxes are lump sum. The first column, in which government spending is wasteful and taxes are treated as lump-sum, shows that four of the seven countries have achieved two-thirds or more of their welfare gains mostly via TFP growth. The exceptions are the three countries that are known to have had low TFP growth over our sample period: Japan, Canada, and Spain. Moving to the case of distortionary taxes raises the TFP contribution (by reducing the factor shares), as does changing the treatment of public spending as optimal rather than wasteful (which raises the growth rate of output, and thus TFP). In the case of optimal spending with distortionary taxes, all countries get a majority of their welfare growth from TFP. Only in Japan and Canada is the contribution of TFP to welfare less than 70 percent, and in most cases it is 75 percent or more.

We check the robustness of the previous results to using a more refined measure of labor input. As noted above, for the main results we use total hours worked as the measure of labor. However, if workers are heterogeneous along dimensions that affect their productivity and are paid different wages as a consequence, we should use a labor input index that recognizes this fact. This amounts to implementing the measure of labor input discussed in section 3.3. The results, for the case of optimal spending with distortionary taxes, are shown in Figure 3. Qualitatively, there is little change. The UK and Spain are still bunched at the top, followed by the US and France. However, with the labor index, Japan drops a little further behind these four leading economies, and ends the period with a cumulated welfare growth rate in between the US and France and the trailing economies, Italy and Canada. Overall, the results look very similar to our baseline case.

Finally, we compare the results we have just obtained using our theory-based welfare metric to those implied by standard proxies for welfare change. In Table 4, we present the average growth rates of GDP and consumption per-capita for our group of countries over our sample period, as well as the average growth rate of our welfare measure under the assumption of optimal government spending and distortionary taxes. First, note the differences in magnitude. Welfare usually grows faster than do conventional measures like consumption per-capita. The difference is typically in the order of one full percentage point per year, which is a striking difference in growth rates. Second, these different measures sometimes produce quite different rankings among countries. Take France, for example. Judged by consumption growth, this country comes at the very bottom of our group of seven countries, significantly behind even Canada. In terms of welfare growth, on the other hand, France comes second.

4.3 Cross Country Results

We now turn to measuring welfare differences across the countries in our sample. For each country and time period, we calculate the welfare gap between that country and the US, as defined in equation (37). Recall that this gap is the loss in welfare of a representative US consumer who is moved permanently to country i starting at time t, expressed as the log gap between the "equivalent consumption" of the consumer in the two cases. In this hypothetical move, the consumer loses the per-capita capital stock of the US, but gains the equivalent capital stock of country i. From time t on, the consumer faces the same product and factor prices and tax rates, and receives the same lump-sum transfers and government expenditure benefits as all the other consumers in country i. In a slight abuse of language, we often use refer to the incremental equivalent consumption as "the welfare difference" or "the welfare gap."

Note that these gaps are all from the point of view of a US consumer. Hence, all the shares in (37), even those used to construct output and TFP growth in country i, are the *US* shares. This naturally raises the question whether our results would be quite different if we took a different country as our baseline. We return to this issue after presenting our basic set of results.[29]

We present numerical results in Table 5. Since the size of the gap varies over time, we present the gap at the beginning of our sample, at the end of our sample, and averaged over the sample period. We present results for three cases: wasteful spending, with lump-sum and distortionary taxes, and optimal spending with distortionary taxes. These numerical magnitudes are useful references in the discussion that follows.

However, the results are easiest to understand in graphical form. We plot the welfare gap for the countries and time periods in our sample in Figures 4 and 5. Note that by definition the gap is zero for the US, since the US consumer neither gains nor loses by moving to the US at any point in time. The vertical axis shows, therefore, the gain to the US consumer of moving to any of the other countries at any point in the sample period, expressed in log points of equivalent consumption. Figure 4 shows the results for the case of wasteful spending and lump-sum taxes. Figure 5 shows the results for our benchmark case, where we allow for distortionary taxes and assume that government expenditure is optimally chosen. Since both figures show qualitatively similar results, for brevity we discuss only the benchmark case.

It is instructive to begin by focusing on the beginning and end of the sample. At the beginning of the sample, expected lifetime welfare in both France and the UK was less than 20 percent lower than in the US (gaps of 16 and 19 percent, respectively). This relatively small gap reflects both the long-run European advantage in leisure and the fact that in the mid-1980s the US was still struggling with its productivity slowdown, while TFP in the leading European economies was growing faster than in the US. Capital accumulation was also proceeding briskly in those countries. By the end of the sample, the continental European economies, Canada and Japan are generally

[29] We conjecture that if we took a second-order approximation to the welfare gap, then the shares in our computation would be averages of the shares in the two countries, and hence bilateral comparisons would be invariant to the choice of a reference country. We leave the investigation of this hypothesis to future research.

falling behind the US, because they had not matched the pickup in TFP growth and investment experienced in the US after 1995. Italy experiences the greatest relative "reversal of fortune," ending up with a welfare gap of nearly 70 percent relative to the US. The results for France are qualitatively similar, but far less extreme. France starts with a welfare gap of 16 percent, and slowly slips further behind, ending with a gap of 21 percent. In continental Europe, only Spain shows convergence to the US in terms of welfare: it starts with a gap of 41 percent, and ends with a gap of 36 percent. However, after 1995 Spain holds steady relative to the US, but does not gain further[30].

The only economy in our sample that exhibits convergence to the US throughout our sample is the UK. Indeed, as Figure 4 shows, under the assumption of wasteful spending and lump-sum taxes, the UK overtakes the US by the end of our sample period. Table 5 shows that in more realistic cases where taxes are assumed to be distortionary the welfare level of the UK is always below that of the US, but the UK shows strong convergence, slicing two-thirds off the welfare gap in two decades in our benchmark case. This result is interesting, because the UK experienced much the same lack of TFP growth in the late 1990s and early 2000s as the major continental European economies.[31] However, the UK had very rapid productivity growth from 1985 to 1995. The other "Anglo-Saxon" country in our sample, Canada, had a welfare level about 30 percent below that of the US in 1985, but the welfare gap had grown by an additional 50 percent by the end of the sample. This result is due primarily to the differential productivity performance of the two countries: TFP in Canada actually fell during the 1990s, and rose only slowly in the early 2000s.

Perhaps the most striking comparison is between the US and Japan. Even in 1985, when its economic performance was the envy of much of the world, Japan was the least attractive country in our sample to an US consumer contemplating emigration; such a consumer would give up nearly 50 percent of his consumption permanently in order to stay in the US instead of moving to Japan. However, like the UK and Spain, Japan was closing the gap with the US until its real estate bubble burst in 1991. The relative performance of the three countries changes dramatically from that point: unlike the UK, which continues to catch up, and Spain, which holds steady, Japan begins to fall behind the US, first slowly and then more rapidly. Having closed to within 43 percent of the US welfare level in 1991, Japan ends our sample 53 percent behind. This cautionary history suggests that it would be interesting to see what the same calculations will show for the US in

[30] Since our open-economy calculations use absorption rather than GDP to construct the Solow residual, one could wonder if Spain's catch up over this period might primarily reflect an unsustainable absorption boom. In our calculations this would have helped narrow the welfare gap vis-a-vis the U.S., but at the cost of putting the country's external accounts in more precarious position than those of other countries. However, inspection of the trends in the ratio of net foreign debt to GDP (drawing from the Lane and Milesi-Ferretti (2007) updated database) suggests that this is not the case: over the period of analysis, the trends for Spain, Italy and the U.S. are all very similar. If one looks instead at the trends in the overall net foreign asset position (comprising not only net debt, but also net equity-related foreign assets), the same conclusion holds roughly until 1998, after which Spain's position deteriorates more rapidly than those of Italy and the U.S. (which follow roughly similar paths throughout, very close also to that followed by the U.K.). Since Spain's partial catch-up to the U.S. in terms of welfare takes place prior to 1995, as noted in the text, this again suggests that the catch-up is not primarily due to a deterioration in the country's external position more acute than those experienced by Italy or the U.S. itself.

[31] For discussion and a suggested explanation, see Basu, Fernald, Oulton and Srinivasan (2004).

another 10 or 15 years, after the bursting of the US real estate bubble and the associated financial crisis.

As we did for the within-country results, we investigate whether the cross-country welfare gaps are driven mostly by the TFP gap or by differences in capital per worker. The results are in Table 6. We focus on the last column of Panel C, which gives results averaged over the full sample period for our baseline case of optimal spending with distortionary taxes. We find that for five of the six countries, TFP is responsible for the vast majority of the welfare gap relative to the US. Indeed, for Japan TFP accounts for more than 100 percent of the gap (meaning that Japan has generally had a higher level of capital per-capita than the US). Thus we arrive at much the same conclusion as Hall and Jones (1999), although our definition of TFP is quite different from the one they used, and we do not focus only on steady-state differences. The exception to this rule is the UK. The average welfare gap between the US and the UK is driven about equally by TFP and by capital. Panel B shows that by the end of the sample, the UK had surpassed the US in "welfare-relevant TFP," and more than 100 percent of the gap was driven by the difference in per-capita capital between the two economies.

We now check the robustness of the preceding results along two dimensions.

First, as noted above, we wish to see whether our welfare rankings among countries is sensitive to the choice of the baseline country. We thus redo the preceding exercises taking France as the baseline country. France is the largest and most successful continental European economy in our sample, and by revealed preference French households place much higher weight on leisure than do US ones.[32] We summarize the results for our baseline case of optimal spending with distortionary taxes in Figure 6. For ease of comparison with the preceding cross-country figures, we still normalize the US welfare level to zero throughout, even though the comparison is done from the perspective of the French consumer and is based on French shares. Reassuringly, we see that the qualitative results are unchanged. France and the UK start closest to the US in 1985, but even they are well behind the US level of welfare. The UK converges towards the US welfare level and so, from a much lower starting point, does Spain. All the other economies, including France, fall steadily farther behind the US over time. Interestingly, from the French point of view almost all the other countries are shifted down vis-a-vis the US relative to the rankings from the US point of view. It appears that the representative French consumer believes that the US is further ahead of France than does the representative US consumer!

Second, we redo the baseline results (from the point of view of the US consumer) using an index of heterogeneous labor input. Our method demands that we construct the labor index for each country weighting the hours of different types of workers by the US shares. However, unlike the within-country case in the previous sub-section, where we used country-specific shares, this procedure yields quite different results than the baseline case using hours. Figure 7 presents the results graphically for our baseline case of optimal spending with distortionary taxes. We still find

[32] As noted above, data limitations prevent us from including Germany in our sample, although it would be another natural baseline.

that the UK and France are closest to the US in welfare levels, but now we no longer find strong evidence of convergence for the UK; both countries steadily fall behind the US over time. Spain shows the greatest difference relative to our previous results. Instead of converging or holding steady relative to the US, Spain falls behind monotonically. This result is driven by the fact that the Spanish *growth rate* of labor input is very high for categories of workers that receive a high *share* of labor income in the US (particularly workers with a "middle" level of education, as defined by EU-KLEMS). As a check, we computed results using the index of labor input, but from the French point of view (i.e., using French shares). We find very similar results for Spain, showing that this result is likely to obtain whenever one applies disaggregated labor shares from rich countries to growth rates of labor input for middle-income (or poor) countries. For this reason, we use total hours as our baseline measure of labor input, in both the within- and cross-country cases.

Finally, as we did for the results on within-country welfare growth, we compare our welfare results to those based on traditional measures, namely PPP-adjusted GDP and consumption per-capita. The results are in Table 7. Focusing on Panel B, for the final year of our sample, we see that the three measures sometimes give identical results. For example, the US is atop the world rankings by all three measures, although the gap between the US and the second-ranked country is much smaller in percentage terms for welfare (6 percent) than it is for the other two variables (18 or 19 percent). On the other hand, the differences can be striking. For example, Canada, which is the second only to the US in GDP and third judged by consumption, is third from the bottom in our welfare ranking. Canada, which leads Spain by 20 percent or more in terms of consumption and GDP per-capita, trails Spain by about 10 percent in our welfare comparison. Indeed, Spain is last within our group of countries in terms of the conventional metrics of consumption and GDP, but ranks fourth in welfare terms, trailing only the US, UK and France. For the other countries, the welfare measure is not so kind. Japan trails the US by only 26 percent in GDP per-capita, but double that—52 percent—in terms of welfare. Similarly, Italy has more than 60 percent of the per-capita GDP of the US, but only about one-third the welfare level. On the other hand, France trails the US by 40 percent in consumption per-capita, but by only half that amount in terms of a welfare. Thus, our measure clearly provides new information on welfare differences among countries.

5 Relationship to the Literature

Measuring welfare change over time and differences across countries using observable national income accounts data has been a long-standing challenge for economists. We note here the similarities and differences between our approach and ones that have been taken before. We also suggest ways in which our results might be useful in other fields of economics, where the same question arises in different contexts.

Nordhaus and Tobin (1972) originated one approach, which is to take a snapshot of the economy's flow output at a point in time and then go "beyond GDP," by adjusting GDP in various

ways to make it a better flow measure of welfare. Nordhaus and Tobin found that the largest gap between flow output and flow welfare comes from the value that consumers put on leisure. Their result motivated us to add leisure to the period utility function in our model, which is standard in business-cycle analysis but not in growth theory. Nordhaus and Tobin's approach has been followed recently by Jones and Klenow (2010) who add other corrections, notably for life expectancy and inequality. However, this point-in-time approach does not take into account the link between today's choices and future consumption or leisure possibilities. For example, high consumption in the measured period might denote either permanently high welfare or low current investment. Low investment would mean that consumption must fall in the future, so its current level would not be a good indicator of long-term welfare. Our approach is to go beyond point-in-time measures of welfare and compute the expected present discounted value of consumers' entire sequence of period utility. In so doing, we also shift the focus from consumers' particular choices of one period's consumption and leisure to their intertemporal choice sets, as defined by their assets and the sequence of prices they face. This approach pays off particularly when we measure welfare differences across countries.

Our intertemporal approach echoes the methods used in the literature started by Weitzman (1976) and analyzed in depth by Weitzman (2003), with notable contributions from many other authors.[33] This literature also relates the welfare of a representative agent to observables; for example, Weitzman (1976) linked intertemporal welfare to net domestic product (NDP). Unlike our model which allows for uncertainty about the future, this literature almost always assumes perfect foresight.[34] As we discuss later, allowing for uncertainty is important when forward-looking rules for measurement are applied to actual data. More importantly, the results in these papers are derived using a number of strong restrictions on the nature of technology (typically an aggregate production function with constant returns to scale), product market competition (always assumed to be perfect), and the allowed number of variables that are exogenous functions of time, such as technology or terms of trade (usually none, but sometimes one or two). Most of the analysis in the literature applies to a closed economy where growth is optimal.[35] Taken together, this long list of assumptions greatly limits the domain of applicability of the results.

By contrast, we derive all our results based only on first-order conditions from household optimization, which allows for imperfect competition in product markets of an arbitrary type and for a vast range of production possibilities, with no assumption that they can be summarized by an aggregate production function or a convex technology set. (This makes it easy to apply our results to modern trade models, for example, since these models often assume imperfect competition with substantial producer heterogeneity.) We do not need to assume that the economy follows an opti-

[33] A far from exhaustive list includes Asheim (1994), Arronson and Löfgren (1995), Mino (2004), Sefton and Weale (2006), Basu, Pascali, Schiantarelli and Serven (2009), and Hulten and Schreyer (2010). Reis (2005) analyses the related problem of computing a dynamic measure of inflation for a long-lived representative consumer.

[34] Arronson and Löfgren (1995) allow for stochastic population growth, and Weitzman (2003, ch. 6) considers shocks coming from stochastic depreciation of capital.

[35] Sefton and Weale (2006) and Hulten and Schreyer (2010) consider an open economy with changes in the terms of trade and Mino (2004) analyses Marshallian spillovers to R&D (all under perfect foresight).

mal growth path. We are also able to allow for a wide range of shocks, including but not limited to changes in technology, tax rates, terms of trade, government purchases, the size of Marshallian spillovers, monetary policy, tariffs, and markups.[36] Crucially, we do not need to specify the sources of structural shocks to the economy. The key to the generality of our results is that we condition on observed prices and asset stocks without needing to model *why* these quantities take on the values that they do.[37] To our knowledge, in the literature started by Weitzman (1976), this paper is the first to produce empirical measures of intertemporal welfare in a framework that allows productivity to vary over time.

Our work clarifies and unifies several results in other literatures, especially international economics. Kohli (2004) shows in a static setting that terms-of-trade changes can improve welfare in open economies even when technology is constant. Kehoe and Ruhl (2008) prove a related result in a dynamic model with balanced trade: opening to trade may increase welfare, even if it does not change TFP. In these models, which assume competition and constant returns, technology is equivalent to TFP. We generalize and extend these results, and show that in a dynamic environment with unbalanced trade welfare can also change if there are changes in the quantity of net foreign assets or in their rates of return.[38] In general, we show that there is a link between observable aggregate data and welfare in an open economy, which is the objective of Bajona, Gibson, Kehoe and Ruhl (2010). While we agree with the conclusion of these authors that GDP is not a sufficient statistic for uncovering the effect of trade policy on welfare, we show that one can construct such a sufficient statistic by considering a relatively small number of other variables. Our results also shed light on the work of Arkolakis, Costinot and Rodriguez-Clare (2011). These authors show that in a class of modern trade models, which includes models with imperfect competition and micro-level productivity heterogeneity, one can construct measures of the welfare gain from trade without reference to micro data. Our results imply that this conclusion actually holds in a much larger class of models, although the exact functional form of the result in Arkolakis et al. (2011) may not. Finally, since changes in net foreign asset positions and their rates of return are extremely hard to measure, we show that one can measure welfare using data only on TFP and the capital stock, even in an open economy, provided that TFP is calculated using absorption rather than GDP as the output concept.

Our work provides a different view of a large and burgeoning literature that investigates the productivity differences across countries. As noted above, if we specialize our cross-country result to

[36] See also Sandleris and Wright (2011) for an attempt to extend the basic ideas in Basu and Fernald (2002) in order to evaluate the welfare effects of financial crises. These papers try to derive methods to measure the welfare effects of a particular shock, which requires specifying an explicit counterfactual path that the economy would have followed in the absence of the shock.

[37] We do need to forecast the present value of future TFP in order to implement our results in data. It is an open question whether specifying a complete general-equilibrium structure for the model would improve our forecasts substantially. We decided that the possible gain in forecasting accuracy from specifying a general-equilibrium structure would not compensate for the loss of generality of our results.

[38] The result that openness does not change TFP may be fragile in models with increasing returns. If opening to trade changes factor inputs, either on impact or over time, then TFP as measured by Solow's residual will change as well, which we show has an effect on welfare even holding constant the terms of trade.

the lump sum-optimal spending case, we obtain something closely related to the results produced by the "development accounting" literature. We show that in that case, (the present value of) the log differences in TFP levels emphasized by the developing accounting literature need to be supplemented with only one additional variable, namely log level gaps in capital per person, in order to serve as a measure of welfare differences among countries.[39] This result implies immediately that estimates of TFP losses due to allocative inefficiency (e.g., Hsieh and Klenow, 2009) can be translated to estimates of welfare losses.

Our results are also related to an earlier literature on "industrial policy." and to more recent literature on the effect of "reallocation". Bhagwati, Ramaswami and Srinivasan (1969) and Bulow and Summers (1986) argue welfare would be enhanced by policies to promote growth in industries where there are rents, for example stemming from monopoly power.[40] The TFP term in our basic result, equation (**??**), captures this effect. When firms have market power, their output grows faster than the share-weighted sum of their inputs, even when their technology is constant. Thus, aggregate TFP rises when firms with above-average market power grow faster than average. At first this result sounds counterintuitive, since it implies that welfare is enhanced by directing more capital and labor to the most distorted sectors. However, the logic is exactly the same as the usual result that firms with the greatest monopoly power should also receive the largest unit subsidies to increase their output.

A number of recent papers suggest that a substantial fraction of output growth within countries comes from reallocation, broadly defined. However, given the different definitions of "reallocation" used in the literature, it is not clear how much reallocation matters for welfare. Our work suggests that the literature should focus on quantifying the increment to aggregate TFP growth from reallocation. Furthermore, it shows that TFP is what matters, not technical efficiency. TFP contributions can come from either higher technical efficiency, by exploiting increasing returns to scale, or by allocating inputs more efficiently across firms. When TFP and technical efficiency diverge, it is TFP that matters for welfare. We have concentrated on aggregate TFP and capital accumulation without asking how individual firms and sectors contribute to these welfare-relevant variables. Domar (1961) and Basu and Fernald (2002) show how one can decompose standard TFP based on GDP into sectoral and firm-level contributions. However, our results above show that in an open economy one should base TFP measures on absorption rather than GDP. There is as yet no parallel to Domar's (1961) decomposition for absorption or net investment, since that would require allocating the output of individual industries to particular components of final expenditure. The approach of Basu, Fernald, Fisher and Kimball (2010), based on the use of input-output tables, may be helpful in this endeavor.

Finally, our work is closely related to the program of developing sufficient statistics for welfare

[39] As we show, what matters is the present discounted value of TFP differences. Moreover, one needs to compute TFP using different shares than the ones used by the development accounting literature, and switch to a different output concept (based on domestic absorption) in an open economy.

[40] A second-best policy might involve trade restrictions to protect such industries from foreign competition. If lump-sum taxes are available, the optimal policy is always to target the distortion directly through a tax-cum-subsidy scheme.

analysis, surveyed by Chetty (2009). We have proposed such a statistic for a representative consumer in a macroeconomic context. As Chetty notes, such measures can be used to evaluate the effects of policies. Suppose that one wishes to evaluate the effect of a policy change—for example, a change in trade policy, as in Kehoe and Ruhl (2010). The usual method is to relate the policy change to a variety of economic indicators, such as GDP, capital accumulation, or the trade balance, and then try to relate the indicators to welfare informally. Our work suggests that one can dispense with these "intermediate targets," and just directly relate the welfare outcome to a change in policy, or to some other shock.

6 Conclusions

We show that aggregate TFP, appropriately defined, and the capital stock can be used to construct sufficient statistics for the welfare of a representative consumer. To a first order approximation, the change in the consumer's welfare is measured by the expected present value of aggregate TFP growth (and its revision) and by the change in the capital stock. This result holds regardless of the type of production technology and the degree of product market competition, and applies to closed or open economies with or without distortionary taxation. Crucially, TFP has to be calculated using prices faced by households rather than prices facing firms. In modern, developed economies with high rates of income and indirect taxation, the gap between household and firm TFP can be considerable. Finally, in an open economy, the change in welfare will also reflect present and future changes in the returns on net foreign assets and in the terms of trade. However, these latter terms disappear if absorption rather than GDP is used as the output concept for constructing TFP, and TFP and the initial capital stock are again sufficient statistics for measuring welfare in open economies. Most strikingly, these variables also suffice to measure welfare level differences among countries, with both variables computed as log level deviations from a reference country.

We extend the existing literature on intertemporal welfare measurement by deriving all our results from household first-order conditions alone. The generality of our derivation allows us to propose a new interpretation of TFP that sheds new light on three distinct areas of study. We show that the measures of cross-country TFP differences produced by the "development accounting" literature are directly relevant for calculating welfare differences among countries. We find that readily-available national accounts data can be used to construct welfare measures for open economies, which can be used to evaluate the effects of trade policies and tariff changes. Finally, we argue that a large productivity literature that measures the effects of "misallocation" or "reallocation" of inputs can relate these results to welfare by connecting input misallocation to changes in aggregate, household-centric TFP.

We illustrate our results by using national accounts data to measure welfare growth rates and gaps in a sample of developed countries. For reasonable assumptions about fiscal policy, we find that over our sample period the UK and Spain are the leaders in welfare growth rates. Throughout our sample period, however, the US is the world leader in welfare levels. The UK converges steadily

towards US levels of welfare, and is within a few percent of catching the US by the end of our sample, 2005. At the start of our sample, several countries show evidence of convergence to the US, but by the end almost all countries are diverging away from US welfare levels. This divergence is particularly stark for Japan and Italy, which end the sample with less than half the per-capita welfare of the US.

References

[1] Arkolakis, C., Costinot, A., Rodríguez-Clare, A., 2009. New Trade Models, Same Old Gains? NBER Working Papers 15628.

[2] Aronsson, T., Löfgren, K. G., 1995. National Product Related Welfare Measurement in the Presence of Technological Change, Externalities and Uncertainty. Environmental and Resource Economics 5, 321–332.

[3] Arrow, K. J. 1962. The Economic Implications of Learning by Doing. Review of Economic Studies 29: 155-73.

[4] Asheim, G. B, 1994. Net National Product as an Indicator of Sustainability. Scandinavian Journal of Economics 96, 257-265.

[5] Bhagwati, J.m Ramaswami, V. K., Srinvasan, T. N., 1969. Domestic Distortions, Tariffs, and the Theory of Optimum Subsidy: Some Further Results. Journal of Political Economy 77, 1005-1010.

[6] Bajona, C., Gibson M., Kehoe T. J., Ruhl K. J., 2010. Trade Liberalization, Growth, and Productivity. Mimeo.

[7] Baker, D., Rosnick, D., 2007. Usable productivity" growth in the US: an international comparison, 1980-2005. International Productivity Monitor.

[8] Barro, R., 1990. Government Spending in a Simple Model of Endogenous Growth. Journal of Political Economy 98, 103-125.

[9] Barro, R. J., Sala-i-Martin, X., 2004. Economic Growth. The MIT Press, Cambridge.

[10] Basu, S., Fernald, J.G., 2002. Aggregate productivity and aggregate technology. European Economic Review 46, 963-991.

[11] Basu, S., Fernald J. G., Oulton, N., Srinivasan, S., 2003. The Case of the Missing Productivity Growth: Or, Does Information Technology Explain why Productivity Accelerated in the US but not the UK? NBER Working Papers 10010.

[12] Basu, S., Pascali, L., Schiantarelli F., Serven, L., 2009. Productivity, Welfare and Reallocation: Theory and Firm-Level Evidence. NBER Working Papers 15579.

[13] Bosca, J. E., Garcia, J., Taguas, D., 2005. Tipos Efectivos de Gravamen y Convergencia Fiscal en la OCDE: 1965-2001. Revista de Economia Publica 174, 119-141.

[14] Bulow, J., Summers, L. H, 1986. A Theory of Dual Labor Markets with Application to Industrial Policy,Discrimination, and Keynesian Unemployment. Journal of Labor Economics 4, 376-414.

[15] Carey, D. and H. Tchilinguirian, 2000. Average effective tax rates on capital, labour and consumption. OECD Economics Department Working Paper 258.

[16] Cooley, T. F., Prescott, E. C., 1995. Economic Growth and Business Cycles in Frontiers of Business Cycle Research, T.F. Cooley Editor, Princeton University, Princeton.

[17] Chetty, R., 2009. Sufficient Statistics for Welfare Analysis: A Bridge Between Structural and Reduced-Form Methods. Annual Review of Economics 1, 451-488.

[18] Domar, E., 1961. On the Measurement of Technological Change. Economic Journal 71, 709-729.

[19] Hall, R., Jones, C. I., 1999. Why Do Some Countries Produce So Much More Output Per Worker Than Others? The Quarterly Journal of Economics 114(1), 83-116.

[20] Hsieh, C. T., Klenow, P., 2009. Misallocation and Manufacturing TFP in China and India. The Quarterly Journal of Economics 124, 1403 -1448.

[21] Hulten, C. R., 1973. Divisia Index Numbers. Econometrica 41, 1017-1026.

[22] Hulten, C.R, 1978. Growth accounting with intermediate inputs, The Review of Economic Studies 45, 511-518.

[23] Hulten, C.R., Schreyer, P., 2010. GDP, Technical Change and the Measurement of Net Income: the Weitzman Model Revisited, NBER Working Paper 16010, May.

[24] Jones, C.I., 1995. R&D-Based Models of Economic Growth, Journal of political Economy, 103, 759-83.

[25] Jones, C. I., Klenow, P. J., 2010. Beyond GDP? Welfare across Countries and Time. NBER Working Papers 16352.

[26] Kehoe, T.J., Ruhl, K.J., 2008. Review of Economic Dynamics 11, 804-819.

[27] Kehoe, T.J., Ruhl, K.J., 2010. Why Have Economic Reforms in Mexico Not Generated Growth? Journal of Economic Literature 48, 1005–1027.

[28] King, R.G., Plosser, C.I., Rebelo, S.T., 1988. Production, growth and business cycles. Journal of Monetary Economics 21, 195-232.

[29] King, R.G., Rebelo, S.T., 1999. Resuscitating Real Business Cycle in Handbook of Macroeconomics, Taylor, J.B and Woodford M. editors , North Holland.

[30] Kohli, U., 2004. Real GDP, Real Domestic Income, and Terms-of-Trade Changes. Journal of International Economics 62, 83-106.

[31] Kremer, M., Onatski, A., Stock J., 2001. Searching for Prosperity. NBER Working Papers 8250.

[32] Lane, P. R., Milesi-Ferretti, G. M., 2001. The external wealth of nations: measures of foreign assets and liabilities for industrial and developing countries. Journal of International Economics 55(2), 263-294.

[33] Lane, P. R., Milesi-Ferretti, G. M., 2007. The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities, 1970–2004. Journal of International Economics 73, 223-250.

[34] Lloyd, P. J., 1977. Two-Stage Expenditure Minimization and some Welfare Applications. Quarterly Journal of Economics 91, 531-554.

[35] Mendoza, E.G., Razin, A., Tesar L. L., 1994. Effective Tax Rates in Macroeconomics, Cross-Country Estimates of Tax Rates on Factor Incomes and Consumption. Journal of Monetary Economics, 34(3), 297-323.

[36] Mino, K., 2004. Weitzman's rule with market distortions. Japan and the World Economy 16, 307-329.

[37] Mulligan, C. B., 2001. Aggregate Implications of Indivisible Labor. The B.E. Journal of Macroeconomics 0, pages 4-16.

[38] Nordhaus, W., Tobin, J., 1972. Is growth obsolete? NBER General Series 96

[39] O'Mahony, M., Timmer, M.P., 2009. Output, Input and productivity measures at the industry level: the EU KLEMS database. Economic Journal 119, 374-403.

[40] Quah, D. T, 1996. Twin Peaks: Growth and Convergence in Models of Distribution Dynamics. Economic Journal 106, 1045-55.

[41] Rebelo, S. , 1991. Long-Run Policy Analysis and Long-Run Growth. Journal of Political Economy 99 (3): 500.

[42] Reis, R., 2005. A Dynamic Measure of Inflation. NBER Working Papers 11746.

[43] Rogerson, R., Wright, R., 1988. Involuntary Unemployment in Economies with Efficient Risk Sharing. Journal of Monetary Economics 22, 501-15.

[44] Romer, P. M., 1986. Increasing Returns and Long Run Growth. Journal of Political Economy, 94, 1002–37.

[45] Rotemberg, J.J., Woodford, M., 1991. Markups and the Business Cycle in NBER Macroeconomics Annual Volume 6 , Blanchard O.J. and Fischer S. editors.

[46] Sandleris, G., Wright, M. J., 2011. The Costs of Financial Crises: Resource Misallocation, Productivity and Welfare in the 2001 Argentine Crisis. NBER Working Papers 217552.

[47] Sefton, J., Weale, M., 2006. The Concept of Income in a General Equilibrium. Review of Economic Studies 73, 219-249.

[48] Serven, L., 1995. Capital Goods Imports, the Real Exchange Rate and the Current Account. Journal of International Economics 39, 79-101.

[49] Solow, R., M., 1957. Technical Change and the Aggregate Production Function. Review of Economics and Statistics 39, 312-320.

[50] Weil, D. , 2008. Economic Growth. Pearson.

[51] Weitzman, M., 1976. On the welfare significance of national product in a dynamic economy. Quarterly Journal of Economics 90, 156-162.

[52] Weitzman, M., 2003. Income capital and the maximum principle. Harvard University Press, Cambridge.

A Appendix: Derivations

A.1 Proposition 1

The representative household maximizes intertemporal utility:

$$W_t = \sum_{s=0}^{\infty} \frac{1}{(1+\rho)^s} \frac{N_{t+s}}{H} U(C_{t+s}; \overline{L} - L_{t+s}) \tag{A.1}$$

under the following budget constraint:

$$P_t^I K_t N_t + B_t N_t = (1-\delta) P_t^I K_{t-1} N_{t-1} + \left(1 + i_t^B\right) B_{t-1} N_{t-1} + P_t^L L_t N_t + P_t^K K_{t-1} N_{t-1} + \Pi_t N_t - P_t^C C_t N_t \tag{A.2}$$

The laws of motion for N_t and for X_t are:

$$N_t = N_0 (1+n)^t \tag{A.3}$$

$$X_t = X_0 (1+g)^t \tag{A.4}$$

while the law of motion for capital is:

$$K_t N_t = (1-\delta) K_{t-1} N_{t-1} + I_t N_t$$

Normalize $H = 1$. For a well defined steady-state in which hours of work are constant we assume that the utility function has the King Plosser and Rebelo form (1988):

$$U(C_{t+s}; L - \overline{L}_s) = \frac{1}{1-\sigma} {C_{t+s}}^{1-\sigma} \nu(\overline{L} - L_{t+s}) \tag{A.5}$$

where $\nu(\overline{L} - L_{t+s})$ is an increasing and concave function of leisure, and assumed to be positive.

We can re-write the intertemporal utility function, the budget constraint and the law of motion for capital in a normalized form as follows:

$$v_t = \sum_{s=0}^{\infty} \beta^s U(c_{t+s}; \overline{L} - L_{t+s}) \tag{A.6}$$

$$k_t + b_t = \frac{(1-\delta) + p_t^K}{(1+g)(1+n)} k_{t-1} + \frac{(1+r_t)}{(1+g)(1+n)} b_{t-1} + p_t^L L_t + \pi_t - p_t^C c_t \tag{A.7}$$

$$k_t = \frac{(1-\delta)}{(1+g)(1+n)} k_{t-1} + i_t \tag{A.8}$$

where: $v_t = \frac{V_t}{X_t^{(1-\sigma)}}$, $c_t = \frac{C_t}{X_t}$ $k_t = \frac{K_t}{X_t}$, $b_t = \frac{B_t}{P_t^I X_t}$, $p_t^K = \frac{P_t^K}{P_t^I}$, $p_t^L = \frac{P_t^L}{P_t^I X_t}$, $p_t^C = \frac{P_t^C}{P_t^I}$, $(1+r_t) = \frac{(1+i_t)}{(1+\pi_t)}$, $\pi_t = \frac{\Pi_t}{P_t^I X_t}$ and $\beta = \frac{(1+n)(1+g)^{1-\sigma}}{(1+\rho)}$.

The Lagrangean for this problem is:

$$
\begin{aligned}
\Lambda_t &= E_t \sum_{s=0}^{\infty} \beta^s \{U(c_{t+s}; \overline{L} - L_{t+s}) \\
&\quad + \lambda_{t+s} \left(-k_{t+s} - b_{t+s} + \frac{(1-\delta) + p^K_{t+s}}{(1+g)(1+n)} k_{t+s-1} + \frac{(1 + r_{t+s})}{(1+g)(1+n)} b_{t+s-1} + p^L_{t+s} L_{t+s} + \pi_{t+s} - p^C_{t+s} c_{t+s} \right)\}
\end{aligned}
$$

The FOCs are:

$$
U_{c_t} - \lambda_t p^C_t = 0 \tag{A.9}
$$

$$
U_{L_t} + \lambda_t p^L_t = 0 \tag{A.10}
$$

$$
-\lambda_t + \beta E_t \frac{(1-\delta) + p^K_{t+1}}{(1+g)(1+n)} \lambda_{t+1} = 0 \tag{A.11}
$$

$$
-\lambda_t + \beta \frac{1}{(1+g)(1+n)} E_t (1 + r_{t+1}) \lambda_{t+1} = 0 \tag{A.12}
$$

Define with $\widehat{x} = \frac{x_t - x}{x}$ the percent deviation from the steady-state of a variable (x is the steady-state value of x_t). Taking a first order approximation of the Lagrangean (which equals the value function along the optimal path), one obtains:

$$
\begin{aligned}
v_t - v &= E_t [\sum_{s=0}^{\infty} \beta^s (U_c c \widehat{c}_{t+s} + U_L L \widehat{L}_{t+s} \\
&\quad + \lambda p^L L \widehat{L}_{i,t+s} - \lambda p^C c \widehat{c}_{t+s} - \lambda k \widehat{k}_{t+s} - \lambda b \widehat{b}_{t+s}) \\
&\quad + \sum_{s=0}^{\infty} \beta^{s+1} (\lambda \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t+s} + \lambda \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t+s}) \\
&\quad + \sum_{s=0}^{\infty} \beta^s \widehat{\lambda}_{t+s} \left(-k - b + \frac{(1-\delta) + p^K}{(1+g)(1+n)} k + \frac{(1+r)}{(1+g)(1+n)} b \right. \\
&\quad + p^L L + \pi - p^C c)] \\
&\quad + \sum_{s=0}^{\infty} \beta^s (\lambda p^L L \widehat{p}^L_{t+s} + \frac{p^K k}{(1+g)(1+n)} \widehat{p}^K_{t+s} - p^C_t c \widehat{p}_{t+s} + \pi \widehat{\pi}_{t+s} + \frac{rb}{(1+g)(1+n)} \widehat{r}_{t+s}) \\
&\quad + \lambda \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t-1} + \lambda \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t-1}
\end{aligned} \tag{A.13}
$$

Using the first order conditions, the first five lines equal zero. Moreover, since we are considering a closed economy case, in equilibrium $b_t = 0$. Hence, we get:

$$
\begin{aligned}
v_t &= v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^L L \widehat{p}_{t+s}^L + \frac{p^K k}{(1+g)(1+n)} \ \widehat{p}_{t+s}^K - p^C c \widehat{p}_{t+s} + \pi \widehat{\pi}_{t+s}) \right] \\
&\quad + \lambda \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t-1} \qquad \text{(A.14)}
\end{aligned}
$$

Now linearize the budget constraint and the law of motion for capital:

$$
\begin{aligned}
& k \ \widehat{k}_t + b \widehat{b}_t - \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t-1} - \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t-1} - p^L L \widehat{L}_t - p^L L \widehat{p}_t^L - \frac{p^K k}{(1+g)(1+n)} \widehat{p}_t^K \\
& - \frac{rb}{(1+g)(1+n)} \widehat{r}_t - \pi \widehat{\pi}_t + p^C c \widehat{c}_t + p^C c \widehat{p}_t = 0 \qquad \text{(A.15)}
\end{aligned}
$$

$$
k \widehat{k}_t = \frac{(1-\delta)}{(1+g)(1+n)} k \widehat{k}_{t-1} + i \widehat{i}_t \qquad \text{(A.16)}
$$

Using these two equations and the steady-state version of the FOC for capital in (A.14) gives us:

$$
\begin{aligned}
v_t &= v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^C c \widehat{c}_{t+s} + i \widehat{i}_{t+s} - \frac{p^K k}{(1+g)(1+n)} \widehat{k}_{t+s-1} - p^L L \widehat{L}_{t+s} \right] \\
&\quad + \lambda \frac{1}{\beta} k \widehat{k}_{t-1} \qquad \text{(A.17)}
\end{aligned}
$$

Take the difference between the expected level of intertemporal utility v_t as in (A.17) and v_{t-1} and use the fact that: $\frac{x_t - x_{t-1}}{x} \simeq \log x_t - x_{t-1}$ for a positive x_t, to get:

$$
\begin{aligned}
\Delta v_t &= E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^C c \log c_{t+s} + i \log i_{t+s} - p^L L \log L_{t+s} - \frac{p^K k}{(1+g)(1+n)} \log \ k_{t+s-1} \right] \\
&\quad - E_{t-1} \sum_{s=0}^{\infty} \beta^s \lambda \left[p^C c \log c_{t+s-1} + i \log i_{t+s-1} - p^L L \log L_{t+s-1} - \frac{p^K k}{(1+g)(1+n)} \log \ k_{t+s-2} \right] \\
&\quad + \lambda \frac{1}{\beta} k \Delta \log k_{t-1} \qquad \text{(A.18)}
\end{aligned}
$$

The right hand side, after adding and subtracting $E_t x_{t+s}$ for each variable x_{t+s}, can be written as:

$$\begin{aligned}
\Delta v_t &= E_t \sum_{s=0}^{\infty} \beta^s \lambda [p^C c \Delta \log c_{t+s} + i \Delta \log i_t - p^L L \Delta \log L_{t+s} - \frac{p^K k}{(1+g)(1+n)} \Delta \log \ k_{t+s-1}] \\
&+ \sum_{s=0}^{\infty} \beta^s \lambda [p^C c \left(E_t \log c_{t+s} - E_{t-1} \log c_{t+s} \right) + i \left(E_t \log i_{t+s} - E_{t-1} \log i_{t+s} \right) \\
&- p^L L E_t (\log L_{t+s} - E_{t-1} \log L_{t+s}) - \frac{p^K k}{(1+g)(1+n)} \left(E_t \log k_{t+s-1} - E_{t-1} \log k_{t+s-1} \right)] \\
&+ \lambda \frac{1}{\beta} k \Delta \log k_{t-1} \qquad \text{(A.19)}
\end{aligned}$$

Define value added growth (at constant shares) as:[41]

$$\Delta \log Y_t = s_C \Delta \log C_t + s_I \Delta \log I_t \qquad \text{(A.20)}$$

which, in normalized variables can be re-written as:

$$\Delta \log y_t = s_C \Delta \log c_t + s_I \Delta \log i_t \qquad \text{(A.21)}$$

Now, insert this into equation (A.19) and divide both terms by $\lambda p^Y y$ to obtain:

$$\begin{aligned}
\frac{v}{\lambda p^Y y} \frac{\Delta v_t}{v} &= E_t \sum_{s=0}^{\infty} \beta^s [\Delta \log y_t - s_L \Delta \log L_{t+s} - s_K \Delta \log \ k_{t+s-1}] \qquad \text{(A.22)} \\
&+ \sum_{s=0}^{\infty} \beta^s \lambda [(E_t \log y_{t+s} - E_{t-1} \log y_{t+s}) \\
&- s_L E_t (\log L_{t+s} - E_{t-1} \log L_{t+s}) - s_K \left(E_t \log k_{t+s-1} - E_{t-1} \log k_{t+s-1} \right)] \\
&+ \frac{1}{\beta} \frac{k}{p^Y y} \Delta \log k_{t-1}
\end{aligned}$$

Notice that by using the definition of v_t and the FOC for c_t, both evaluated in the steady state, we obtain:

$$\frac{v}{\lambda} = \frac{c p^c}{(1-\beta)(1-\sigma)} \qquad \text{(A.23)}$$

Alternatively, we can measure changes in welfare in terms of changes in equivalent per-capita

[41] Here we are departing slightly from convention, as value added is usually calculated with time varying shares.

consumption. Equivalent consumption C_t^* is implicitly defined by the following equation:

$$\begin{aligned} V_t &= \sum_{s=0}^{\infty} \frac{(1+n)^s}{(1+\rho)^s} \left(C_t^*(1+g)^s\right)^{1-\sigma} \nu(\overline{L}-L)) \\ &= \frac{1}{(1-\sigma)(1-\beta)} C_t^{*1-\sigma} \nu(\overline{L}-L) \end{aligned} \tag{A.24}$$

which can be re-written in terms of normalized variables as:

$$v_t = \frac{1}{(1-\sigma)(1-\beta)} c_t^{*1-\sigma} \nu(\overline{L}-L) \tag{A.25}$$

Linearize this equation to get:

$$v_t - v = \frac{\nu(\overline{L}-L)}{(1-\sigma)(1-\beta)} c^{*-\sigma}(c_t^* - c^*) \tag{A.26}$$

which, together with equation (A.23), implies that:

$$\frac{v}{\lambda p^Y y} \frac{\Delta v_t}{v} = \frac{s_C}{1-\beta} \frac{\Delta c_t^*}{c^*} \tag{A.27}$$

Using the result above into equation (A.22) and noting that $\frac{\Delta c_t^*}{c^*} \simeq \Delta \log c_t^*$, we obtain, to a first order approximation:

$$\begin{aligned} \frac{s_C}{1-\beta} \Delta \log c_t^* &= E_t \sum_{s=0}^{\infty} \beta^s [\Delta \log y_t - s_L \Delta \log L_{t+s} - s_K \Delta \log \ k_{t+s-1}] \\ &+ \sum_{s=0}^{\infty} \beta^s \lambda [(E_t \log y_{t+s} - E_{t-1} \log y_{t+s}) \\ &- s_L E_t (\log L_{t+s} - E_{t-1} \log L_{t+s}) - s_K \left(E_t \log k_{t+s-1} - E_{t-1} \log k_{t+s-1}\right)] \\ &+ \frac{1}{\beta} \frac{k}{p^Y y} \Delta \log k_{t-1} \end{aligned} \tag{A.28}$$

In order to express this result in un-normalized variables notice that:

$$\Delta \log y_t = \Delta \log Y_t - g \tag{A.29}$$

$$\Delta \log k_t = \Delta \log K_t - g \tag{A.30}$$

$$\Delta \log c_t^* = \Delta \log C_t^* - g \tag{A.31}$$

Using equations (A.29)-(A.31), we can rewrite equation (A.22) as:

$$
\begin{aligned}
\frac{s_C}{1-\beta}\Delta\log C_t^* &= E_t\sum_{s=0}^{\infty}\beta^s\Delta\log PR_{t+s} \\
&+\sum_{s=0}^{\infty}\beta^s\left[E_t\log PR_{t+s}-E_{t-1}\log PR_{t+s}\right] \\
&+\frac{1}{\beta}\frac{k}{p^Y y}\Delta\log\frac{K_{t-1}}{N_{t-1}} \\
&+\frac{s_C}{1-\beta}g-\frac{1}{(1-\beta)}g(1-s_K)-\frac{1}{\beta}\frac{k}{p^Y y}g
\end{aligned} \tag{A.32}
$$

where

$$
\log PR_{t+s}\equiv s_C\log C_t+s_I\log I_t-s_L\log L_{t+s}-s_K\log K_{t+s-1}, \tag{A.33}
$$

and $E_t\log PR_{t+s}-E_{t-1}\log PR_{t+s}$ represents the revision in expectations of the level of the productivity residual based on the new information received between t-1 and t. Using equations (A.11) and (A.8) evaluated at the steady-state, one can easily show that $\frac{s_C}{(1-\beta)}-\frac{1}{(1-\beta)}(1-s_K)-\frac{1}{\beta}\frac{k}{p^Y y}=0$, so that the last line in equation (A.32) equals zero. Multiplying both sides of (A.32) by $\frac{1-\beta}{s_C}$ yields equation (7) in the text.

A.2 Proposition 2

Consider the maximization problem of a fictitious household, having the same preferences of an household living in country j and facing prices and per-capita endowments of an household living in country i.

He maximizes the utility function:

$$
\widetilde{W}_t^i=E_t\sum_{s=0}^{\infty}\frac{1}{(1+\rho)^s}\frac{N_{t+s}}{H}U(\widetilde{C^i}_{t+s};\overline{L}-\widetilde{L}_{t+s}^i) \tag{A.34}
$$

facing the following budget constraint:

$$
\begin{aligned}
P_{t+s}^{iI}\widetilde{K}_{t+s}^iN_{t+s}+\widetilde{B}_{t+s}^iN_{t+s} &= \left(1-\delta^j\right)P_{t+s}^{iI}\widetilde{K}_{t+s-1}^iN_{t+s-1}+\left(1+i_t^{B,i}\right)\widetilde{B}_{t+s-1}^iN_{t+s-1}+P_{t+s}^{iL}\widetilde{L}_{t+s}^iN_{t+s} \\
&+P_{t+s}^{iK}\widetilde{K}_{t+s-1}^iN_{t+s-1}+\Pi_{t+s}^iN_{t+s}-P_{t+s}^{iC}\widetilde{C}_{t+s}^iN_{t+s}
\end{aligned} \tag{A.35}
$$

where $\widetilde{}$ variables denote the (unobservable) quantities that the household would choose when facing prices and initial conditions of country i. To simplify the notation in this proof, all variables without the superscript i denote utility, preference parameters, quantities and prices in country j.

Again denote $\widetilde{V}_t^i=\frac{\widetilde{W}_t^i}{N_t}$, per-capita utility. We can re-write the intertemporal utility function and the budget constraint in the normalized form in the following manner:

$$\widetilde{v}_t^i = E_t \sum_{s=0}^{\infty} \beta^s \frac{\widetilde{c}_{t+s}^{i}{}^{1-\sigma} \nu(\overline{L} - \widetilde{L}_{t+s}^i)}{1-\sigma} \tag{A.36}$$

and:

$$\widetilde{k}_{t+s}^i + \widetilde{b}_{t+s}^i = \frac{(1-\delta) + p_{t+s}^{iK}}{(1+g)(1+n)} \widetilde{k}_{t+s-1}^i + \frac{\left(1+r_t^i\right)}{(1+g)(1+n)} \widetilde{b}_{t+s-1}^i + p_{t+s}^{iL} \widetilde{L}_{t+s}^i + \pi_{t+s}^i - p_{t+s}^{iC} \widetilde{c}_{t+s}^i \tag{A.37}$$

where: $\widetilde{v}_t = \frac{\widetilde{V}_t}{X_t^i}$, $\widetilde{c}_{t+s}^i = \frac{\widetilde{C}_{t+s}^i}{X_{t+s}^i}$, $\widetilde{k}_t^i = \frac{K_t^i}{X_t^i}$, $\widetilde{b}_t^i = \frac{B_t^i}{P_t^{iI} X_t^i}$, $p_t^{iK} = \frac{P_t^{iK}}{P_t^{iI}}$, $p_t^{iL} = \frac{P_t^{iL}}{P_t^{iI} X_t^i}$, $p_t^{iC} = \frac{P_t^{iC}}{P_t^{iI}}$, $(1+r_t) = \frac{\left(1+i_t^{B,i}\right)}{\left(1+\pi_t^i\right)}$ and $\pi_t^i = \frac{\Pi_t^i}{P_t^{iI} X_t^i}$. Notice that $\widetilde{v}_t(p^L, p^K, \pi, p^C, (1+r), b, k) = v$. Linearizing $\widetilde{v}_t^i$ around v and using the envelope theorem (and some algebra), we get:

$$\frac{v}{\lambda p^y y} \frac{\widetilde{v}_t^i - v}{v} = E_t \sum_{s=0}^{\infty} \beta^s [s_L \widehat{p}_{t+s}^{iL} + s_K \ \widehat{p}_{t+s}^{iK} \qquad + s_\pi \widehat{\pi}_{t+s}^i - s_C \widehat{p}_{t+s}^{iC} + s_B \widehat{r}_{t+s}^i] \tag{A.38}$$

$$+ \frac{1}{\beta} \frac{k}{p^y y} \widehat{k}_{t-1}^i + \frac{1}{\beta} \frac{b}{p^y y} \widehat{b}_{t-1}^i \tag{A.39}$$

Now, linearize the budget constraint and the law of motion for capital.

$$\begin{aligned} 0 = {} & k \ \widehat{\widetilde{k}}^i{}_{t+s} + b\widehat{\widetilde{b}}^i{}_{t+s} - \frac{(1-\delta) + p^K}{(1+g)(1+n)} k\widehat{\widetilde{k}}^i{}_{t+s-1} - \frac{(1+r)}{(1+g)(1+n)} b\widehat{\widetilde{b}}^i{}_{t+s-1} \\ & - p^L L \widehat{\widetilde{L}}^i{}_{t+s} - p^L L \widehat{p}_{t+s}^{iL} - \frac{p^K k}{(1+g)(1+n)} \widehat{p}_{t+s}^{iK} - \pi \widehat{\pi}_{t+s}^i + p^C c \widehat{\widetilde{c}}^i{}_{t+s} + p^C c \widehat{p}_{t+s}^{iC} \end{aligned} \tag{A.40}$$

$$k\widehat{\widetilde{k}}^i{}_t = \frac{(1-\delta)}{(1+g)(1+n)} k\widehat{\widetilde{k}}^i{}_{t-1} + i\widehat{\widetilde{i}}^i{}_t \tag{A.41}$$

Using the two equations above into equation (A.38), together with the fact that in t-1, $\widehat{k}_{t-1}^i = \widehat{\widetilde{k}}_{t-1}$, we obtain:

$$\frac{v}{\lambda p^y y} \frac{\widetilde{v}_t^i - v}{v} = E_t \sum_{s=0}^{\infty} \beta^s \left[s_C \widehat{\widetilde{c}}^i{}_{t+s} + s_I \widehat{\widetilde{i}}^i{}_{t+s} - s_L \ \widehat{\widetilde{L}}^i{}_{t+s} - s_K \ \widehat{\widetilde{k}}^i{}_{t+s-1} \right] + \frac{1}{\beta} \frac{k}{p^y y} \widehat{k}_{t-1}^i \tag{A.42}$$

Now consider the budget constraint of an household in country i, linearized around country j steady state.

$$k\ \widehat{k^i}_t + b\widehat{b^i}_t - \frac{(1-\delta)+p^K}{(1+g)(1+n)}k\widehat{k^i}_{t-1} - \frac{(1+r)}{(1+g)(1+n)}b\widehat{b^i}_{t-1} - p^L L\widehat{L^i}_t - p^L L\widehat{p}_t^{iL} - \frac{p^K k}{(1+g)(1+n)}\widehat{p}_t^{iK}$$
$$-\pi\widehat{\pi}_t + p^C c\widehat{c^i}_t + p^C c\widehat{p}_t^{iC} = 0 \tag{A.43}$$

where $k_t^i = \frac{K_t^i}{X_t^i}$, $b_t^i = \frac{B_t^i}{P_t^{iI}X_t^i}$, $\pi_t^i = \frac{\Pi_t^i}{P_t^{iI}X_t^i}$ and $p^{iL} = \frac{P_t^{iL}}{P_t^{iI}X_t^i}$. Putting together the two budget constraints in equations (A.40) and (A.43), we get:

$$k\ \widehat{\widetilde{k}^i}_t + b\widehat{\widetilde{b}^i}_t - p^L L\widehat{\widetilde{L}^i}_t + p^C c\widehat{\widetilde{c}^i}_t = k\ \widehat{k^i}_t + b\widehat{b^i}_t p^L L\widehat{L^i}_t + p^C c\widehat{c^i}_t \tag{A.44}$$

which implies that equation (A.42) can be re-written as::

$$\frac{v}{\lambda p^y y}\frac{\widetilde{v}_t^i - v}{v} = E_t\sum_{s=0}^{\infty}\beta^s\left[s_C\widehat{c}_{t+s}^i + s_I\widehat{i}_{t+s}^i - s_L\ \widehat{L}_{t+s}^i - s_K\ \widehat{k}_{t+s-1}^i\right] + \frac{1}{\beta}\frac{k}{p^y y}\widehat{k}_{t-1}^i \tag{A.45}$$

We can get in a similar fashion, the following equation:

$$\frac{v}{\lambda p^y y}\frac{v_t - v}{v} = E_t\sum_{s=0}^{\infty}\beta^s\left[s_C\widehat{c}_{t+s} + s_I\widehat{i}_{t+s} - s_L\ \widehat{L}_{t+s} - s_K\ \widehat{k}_{t+s-1}\right] + \frac{1}{\beta}\frac{k}{p^y y}\widehat{k}_{t-1} \tag{A.46}$$

Using the fact that $\frac{\widetilde{x}_t^i - x}{x} \simeq \log\widetilde{x}_t^i - \log x$ for a generic non-negative variable x, and subtracting equation (A.46) from equation (A.45), we get:

$$\begin{aligned}\frac{v}{\lambda p^y y}\frac{\widetilde{v}_t^i - v}{v} \quad = \quad & E_t\sum_{s=0}^{\infty}\beta^s[s_c(\log c_{t+s}^i - \log c_{t+s}) + s_i(\log i_{t+s}^i - \log i_{t+s}) \\ & -s_L\ (\log L_{t+s}^i - \ln L_{t+s}) - s_K(\log k_{t+s-1}^i - \log k_{t+s})] \\ & +\frac{1}{\beta}\frac{k}{p^Y y}(\log k_{t-1}^i - \log k_{t-1})\end{aligned} \tag{A.47}$$

Using equation (A.27) and the fact that $\frac{s_C}{(1-\beta)} - \frac{1}{(1-\beta)}(1-s_K) - \frac{1}{\beta}\frac{k}{p^Y y} = 0$, after some algebra we can show that:

$$\log\widetilde{C}_t^{*,i} - \log C_t^{*,j} = \frac{(1-\beta^j)}{s_c^j}\left[E_t\sum_{s=0}^{\infty}(\beta^j)^s\left(\log\overline{PR}_{t+s}^i - \log PR_{t+s}^j\right) + \frac{1}{\beta^j}\left(\frac{P^{I,j}K^j}{P^{Y,j}Y^j}\right)\left(\log K_{t-1}^i - \log K_{t-1}^j\right)\right] \tag{A.48}$$

with:

$$\log\ \overline{PR}_{t+s}^i = \left(s_C^j\log C_{t+s}^i + s_I^j\log I_{t+s}^i\right) - s_L^j\log L_{t+s}^i - s_K^j\log K_{t+s-1}^i \tag{A.49}$$

and:

$$\log PR^j_{t+s} = \left(s^j_C \log C^j_{t+s} + s^j_I \log I^j_{t+s}\right) - s^j_L \log L^i_{t+s} - s^j_K \log K^i_{t+s-1} \tag{A.50}$$

where we have reintroduced superscript j to denote equivalent consumption, preference parameters, shares and quantities of country j. These are equations (13), (14) and (15) in the text.

A.3 Proposition 1'

We now show that our method of using TFP to measure welfare can be extended to allow for the presence of taxes and government expenditure, multiple types of goods and labor, and an open economy setting. Instead of having a single general proof, we will proceed by allowing one of these extensions at a time and show how the proof of Proposition 1 needs to be modified in each case in order to yield its generalization, Proposition 1'.

A.3.1 Distortionary Taxes

We now allow for distortionary taxes on capital, labor and financial assets, and for indirect taxes on consumption and investment (at rates τ^K_t,τ^K_t,τ^R_t,τ^C_t,τ^I_t respectively). Let $P^{C\prime}_t$ and $P^{I\prime}_t$ respectively denote the pre-tax prices of consumption and capital goods, so that the tax-inclusive prices faced by the consumer are $P^C_t \equiv P^{C\prime}_t\left(1+\tau^C_t\right)$ and $P^I_t \equiv P^{I\prime}_t\left(1+\tau^I_t\right)$. The representative household's budget constraint now is

$$\begin{aligned} P^{I\prime}_t\left(1+\tau^I_t\right)K_tN_t + B_tN_t &= (1-\delta)P^{I\prime}_t\left(1+\tau^I_t\right)K_{t-1}N_{t-1} + \left(1+i^B_t\left(1-\tau^R_t\right)\right)B_{t-1}N_{t-1} \\ &\quad + P^L_t\left(1-\tau^L_t\right)L_tN_t + P^K_t\left(1-\tau^K_t\right)K_{t-1}N_{t-1} + \Pi_tN_t - P^{C\prime}_t\left(1+\tau^C_t\right)C_tN_t \end{aligned} \tag{A.51}$$

Taking P^I_t as numeraire, the budget constraint can then be re-written in terms of normalized variables as:

$$k_t + b_t = \frac{(1-\delta)+p^K_t\left(1-\tau^K_t\right)}{(1+g)(1+n)}k_{t-1} + \frac{\left(1+r_t\left(1-\tau^R_t\right)\right)}{(1+g)(1+n)}b_{t-1} + p^L_t\left(1-\tau^L_t\right)L_t + \pi_t - p^C_t c_t \tag{A.52}$$

Linearizing it, one obtains:

$$\begin{aligned} 0 \;=\; & k\,\widehat{k}_t - \frac{(1-\delta)+p^K\left(1-\tau^K\right)}{(1+g)(1+n)}k\widehat{k}_{t-1} - \frac{p^K\left(1-\tau^K\right)k}{(1+g)(1+n)}\widehat{p}^K_t + \frac{p^K\tau^Kk}{(1+g)(1+n)}\widehat{\tau}^K_t \\ & - p^L\left(1-\tau^L\right)L\widehat{L}_t - p^L\left(1-\tau^L\right)L\widehat{p}^L_t + p^L\tau^LL\widehat{\tau}^L_t \\ & + p^Cc\widehat{c}_t + p^Cc\widehat{p}^C_t - \pi\widehat{\pi}_t \\ & + b\widehat{b}_t - \frac{\left(1+r\left(1-\tau^R\right)\right)b}{(1+g)(1+n)}\widehat{b}_{t-1} - \frac{r\left(1-\tau^R\right)b}{(1+g)(1+n)}\widehat{r}_t - \frac{r\tau^Rb}{(1+g)(1+n)}\widehat{\tau}^R_t \end{aligned} \tag{A.53}$$

Linearizing the maximization problem as before and using the fact that in equilibrium $B_t = 0$, we get:

$$\begin{aligned}
v_t &= v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^L \left(1-\tau^L\right) L \widehat{p}^L_{t+s} + \frac{p^K \left(1-\tau^K\right) k \; \widehat{p}^K_{t+s}}{(1+g)(1+n)} \; + \pi \widehat{\pi}_{t+s} - p^C c \widehat{p}_{t+s} \right] \\
&\quad - E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^L \tau^L L \widehat{\tau}^L_{t+s} + \frac{p^K \tau^K k}{(1+g)(1+n)} \; \widehat{\tau}^K_{t+s} \right] \\
&\quad + \lambda \frac{(1-\delta) + p^K \left(1-\tau^K\right)}{(1+g)(1+n)} k \widehat{k}_{t-1} \qquad \text{(A.54)}
\end{aligned}$$

Using the linearized budget constraint in the equation above, we obtain:

$$\begin{aligned}
v_t &= v + \lambda E_t \sum_{s=0}^{\infty} \beta^s [\left(1+\tau^C\right) p^C c \widehat{c}_{t+s} + k \; \widehat{k}_{t+s} - \frac{(1-\delta) + p^K \left(1-\tau^K\right)}{(1+g)(1+n)} k \widehat{k}_{t+s-1} - \left(1-\tau^L\right) \; p^L L \widehat{L}_{t+s}] + \\
&\quad + \lambda \frac{(1-\delta) + p^K \left(1-\tau^K\right)}{(1+g)(1+n)} k \widehat{k}_{t-1} \qquad \text{(A.55)}
\end{aligned}$$

Rearranging the terms, we get:

$$\begin{aligned}
v_t &= v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^C c \widehat{c}_{t+s} + k \; \widehat{k}_{t+s} - \frac{(1-\delta) + p^K \left(1-\tau^K\right)}{(1+g)(1+n)} k \widehat{k}_{t+s-1} - \left(1-\tau^L\right) \; p^L L \widehat{L}_{t+s} \right] \\
&\quad + \lambda \frac{1}{\beta} k \widehat{k}_{t-1} \qquad \text{(A.56)}
\end{aligned}$$

Using this last equation together with the linearized law of motion for capital:

$$k \widehat{k}_{t+s} - \frac{(1-\delta)}{(1+g)(1+n)} k \widehat{k}_{t+s-1} = i \widehat{i}_{t+s} \qquad \text{(A.57)}$$

into equation (A.56), we obtain:

$$\begin{aligned}
v_t &= v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^C \left(1+\tau^C\right) c \widehat{c}_{t+s} + i \widehat{i}_{t+s} - \; p^L \left(1-\tau^L\right) L \widehat{L}_{t+s} - \frac{p^K \left(1-\tau^K\right)}{(1+g)(1+n)} k \widehat{k}_{t+s-1} \right] \\
&\quad + \lambda \frac{1}{\beta} k \widehat{k}_{t-1} \qquad \text{(A.58)}
\end{aligned}$$

With the consumption and investment shares of GDP defined using tax-inclusive prices, we still have

$$\Delta \log Y_t = s_C \Delta \log C_t + s_I \Delta \log I_t \qquad \text{(A.59)}$$

or, in terms of normalized variables,

$$\Delta \log y_t = s_C \Delta \log c_t + s_I \Delta \log i_t \tag{A.60}$$

Using this result and some algebra, equation (A.58) becomes:

$$\begin{aligned}
\frac{v}{\lambda p^Y y}\frac{\Delta v_t}{v} \quad = \quad & E_t \sum_{s=0}^{\infty} \beta^s [\Delta \log y_t - \left(1-\tau^L\right) s_L \Delta \log L_{t+s} - \left(1-\tau^K\right) s_K \Delta \log \ k_{t+s-1}] \\
& + \sum_{s=0}^{\infty} \beta^s \lambda [(E_t \log y_{t+s} - E_{t-1} \log y_{t+s}) \\
& - s_L E_t (\log L_{t+s} - E_{t-1} \log L_{t+s}) - s_K \left(E_t \log k_{t+s-1} - E_{t-1} \log k_{t+s-1}\right)] \\
& + \frac{1}{\beta}\frac{k}{p^Y y} \Delta \log k_{t-1} \qquad (A.61)
\end{aligned}$$

The rest of the proof parallels the one in the subsection A1 and gives the following result:

$$\Delta \log \left(C_t\right)^* = \frac{(1-\beta)}{s_C}\left[E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} + \sum_{s=0}^{\infty} \beta^s \Delta E_t \log PR_{t+s} + \frac{1}{\beta}\left(\frac{P^I K}{P^Y Y}\right) \Delta \log K_{t-1}\right] \tag{A.62}$$

where productivity change is now defined as

$$\Delta \log PR_{t+s} = s_C \Delta \log C_{t+s} + s_I \Delta \log I_{t+s} - \left(1-\tau^L\right) s_L \Delta \log L_{t+s} - \left(1-\tau^K\right) s_K \Delta \log K_{t+s-1} \tag{A.63}$$

A.3.2 Government Expenditure

Assume that government spending takes the form of public consumption valued by consumers. We rewrite the instantaneous utility function as

$$U(C_{t+s}, C_{G,t+s}, L_{t+s}) = \frac{1}{1-\sigma} C(C_{t+s}; C_{G,t+s})^{1-\sigma} \nu(\overline{L} - L_{t+s}) \tag{A.64}$$

where C_G denotes per-capita public consumption and $C(.)$ is homogenous of degree one in its arguments. Government expenditure is financed through a lump-sum tax, so that the budget constraint is now:

$$P_t^I K_t N_t + B_t N_t = (1-\delta) P_t^I K_{t-1} N_{t-1} + \left(1+i_t^B\right) B_{t-1} N_{t-1} + P_t^L L_t N_t + P_t^K K_{t-1} N_{t-1} - P_t^C C_t N_t + \Pi_t N_t - T_t N_t \tag{A.65}$$

where T_t is the per-capita lump-sum tax. Re-writing the household maximization problem in normalized variables and proceeding in a similar fashion with respect to the benchmark case, we get:

$$v_t = v + \lambda E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{U_{c_G} c_G \widehat{c}_{G,t+s}}{\lambda} + p^C c\widehat{c}_{t+s} + i\widehat{i}_{t+s} - \ p^L L\widehat{L}_{t+s} - \frac{p^K k}{(1+g)(1+n)} \widehat{k}_{t+s-1} \right] + \lambda \frac{1}{\beta} k\widehat{k}_{t-1} \quad \text{(A.66)}$$

where $c_{G,t} = \frac{C_{G,t}}{X_t}$. In the presence of public expenditure, the log-change in per-capita GDP is defined as:

$$\Delta \log Y_t = s_C \Delta \log C_t + s_{c_G} \Delta \log C_{G,t} + s_I \Delta \log I_t \quad \text{(A.67)}$$

which, in normalized variables can be re-written as:

$$\Delta \log y_t = s_C \Delta \log c_{t+s} + s_{c_G} \Delta \log c_{G,t} + s_I \Delta \log i_t \quad \text{(A.68)}$$

where s_{c_G} is the steady state value of $s_{c_G,t} = \frac{P_t^G C_{G,t}}{P_t^Y Y_t}$ and P^G is the public consumption deflator.

Using this result and some algebra, equation (A.66) becomes:

$$\begin{aligned} \frac{v}{\lambda p^Y y} \frac{\Delta v_t}{v} = \ & E_t \sum_{s=0}^{\infty} \beta^s [\Delta \log y_t - s_L \Delta \log L_{t+s} - s_K \Delta \log \ k_{t+s-1} + \left(s_{c_G}^* - s_{c_G}\right) \Delta \log c_{G,t}] \quad \text{(A.69)} \\ & + \sum_{s=0}^{\infty} \beta^s \lambda [(E_t \log y_{t+s} - E_{t-1} \log y_{t+s}) \\ & - s_L E_t (\log L_{t+s} - E_{t-1} \log L_{t+s}) - s_K \left(E_t \log k_{t+s-1} - E_{t-1} \log k_{t+s-1}\right)] \\ & + \frac{1}{\beta} \frac{k}{p^Y y} \Delta \log k_{t-1} \end{aligned}$$

where $s_{c_G}^*$ is the steady state value of $s_{c_G,t}^* = \frac{U_{c_G,t} c_{G,t}}{\lambda_t}$. From this point, the algebra is very similar as in the benchmark case and gives the following result:

$$\Delta \log (C_t)^* = \frac{(1-\beta)}{(s_C + s_{C_G}^*)} \left[E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} + \sum_{s=0}^{\infty} \beta^s \Delta E_t \log PR_{t+s} + \frac{1}{\beta} \left(\frac{P^I K}{P^Y Y} \right) \Delta \log K_{t-1} \right] \quad \text{(A.70)}$$

where:

$$\Delta \log PR_{t+s} = \Delta \log Y_{t+s} - s_L \Delta \log L_{t+s} - s_K \Delta \log K_{t+s-1} + \left(s_{c_G}^* - s_{c_G}\right) \Delta \log C_{G,t+s} \quad \text{(A.71)}$$

A.3.3 Multiple Types of Labor, Consumption and Investment Goods

The extension to the case of multiple types of labor, capital and consumption goods is immediate. For simplicity, we could assume that each individual is endowed with the ability to provide different types of labor services, $L_{h,t}$ and that the utility function can be written as:

$$U(C_{1,t+s}, .., C_{Z,t+s}, L_{1,t+s}, ..., L_{H_L,t+s}) = \frac{1}{1-\sigma} C(C_{1,t+s}, .., C_{Z,t+s})^{1-\sigma} \nu \left[\overline{L} - L(L_{1,t+s}, ..., L_{H_L,t+s}) \right] \tag{A.72}$$

where $L(.)$ and $C(.)$ are homogenous functions of degree one, H_L is the number of types of labor and Z is the number of consumption goods. Denote the payment to a unit of $L_{h,t}$, $P_t^{L_h}$.[42]

The budget constraint is now:

$$\begin{aligned} P_t^I K_t N_t + B_t N_t &= (1-\delta) P_t^I K_{t-1} N_{t-1} + \left(1 + i_t^B\right) B_{t-1} N_{t-1} \\ &+ \sum_{h=1}^{H_L} P_t^{L_h} L_{h,t} N_t + P_t^I K_{t-1} N_{t-1} + \Pi_t N_t - \sum_{h=1}^{Z} P_{t+s}^{C_h} C_{h,t+s} N_t \end{aligned} \tag{A.73}$$

Similarly, assume that consumers can purchase H_I different types of investment goods $I_{h,t}$ at prices $P_t^{I_h}$, and combine them into capital. The law of motion of capital is now:

$$K_t N_t = (1-\delta) K_{t-1} N_{t-1} + I_t N_t \tag{A.74}$$

where I_t is a Divisia index where different types of investment goods are aggregated using steady-state shares:

$$I_t = \sum_{h=1}^{H_K} s_{I_h} I_{h,t} \tag{A.75}$$

where s_{I_h} is the steady-state value of $s_{I_{h,t}} \equiv \frac{P_h^I I_{h,t}}{\sum_{z=1}^{H_K} P_{z,t}^I I_{z,t}}$.

Take capital good as the numeraire. Re-writing the household maximization problem in normalized variables and proceeding in a similar fashion with respect to the benchmark case, we get:

$$\begin{aligned} v_t &= v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[\sum_{h=1}^{H_K} p_h^L L_h \ \widehat{p}_{h,t+s}^L + \frac{p^K k}{(1+g)(1+n)} \ \widehat{p}_{t+s}^K - \sum_{i=1}^{Z} p_i^C c_i \widehat{p}_{i,t+s}^C + \pi \widehat{\pi}_{t+s}) \right] \\ &+ \lambda \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t-1} \end{aligned} \tag{A.76}$$

Use the linearized version of equations (A.73)-(A.75) into the equation above, to get:

[42] We assume that the nature of the utility function is such that positive quantities of all types of labors are supplied.

$$v_t = v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[\sum_{i=1}^{Z} p_i^C c_i \widehat{c}_{i,t+s} + \sum_{h=1}^{H_K} p_h^I i_h \widehat{i}_{h,t+s} - \sum_{h=1}^{H_K} p_h^L L_h \ \widehat{L}_{h,t+s} - \frac{p^K k}{(1+g)(1+n)} \ \widehat{k}_{h,t+s-1} \right] + \lambda \frac{1}{\beta} k \widehat{k}_{t-1} \tag{A.77}$$

The log-change in per-capita GDP is now defined as:

$$\Delta \log Y_t = \sum_{i=1}^{Z} s_{C_i} \Delta \log C_{i,t} + \sum_{h=1}^{H_K} s_{i_h} \Delta \log I_{h,t} \tag{A.78}$$

which, in normalized variables can be re-written as:

$$\Delta \log y_t = \sum_{i=1}^{Z} s_{C_i} \Delta \log c_{i,t} + \sum_{h=1}^{H_K} s_{i_h} \Delta \log i_{h,t} \tag{A.79}$$

Using the equation above into (A.77), we get:

$$\begin{aligned} \frac{v}{\lambda p^Y y} \frac{\Delta v_t}{v} = & E_t \sum_{s=0}^{\infty} \beta^s [\Delta \log y_t - \sum_{h=1}^{H_L} s_{L_h} \Delta \log L_{h,t+s} - s_K \Delta \log \ k_{h,t+s-1}] \\ & + \sum_{s=0}^{\infty} \beta^s \lambda [(E_t \log y_{t+s} - E_{t-1} \log y_{t+s}) \\ & - \sum_{h=1}^{H_L} s_{L_{h,t}} E_t (\log L_{h,t+s} - E_{t-1} \log L_{h,t+s}) - (E_t \log k_{t+s-1} - E_{t-1} \log k_{t+s-1})] \\ & + \frac{1}{\beta} \frac{p^I k}{\lambda p^Y y} \Delta \log k_{t-1} \end{aligned} \tag{A.80}$$

The rest of the algebra parallels the benchmark case and gives the following result:

$$\Delta \log (C_t)^* = \frac{(1-\beta)}{s_C} \left[E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} + \sum_{s=0}^{\infty} \beta^s \Delta E_t \log PR_{t+s} + \frac{1}{\beta} \left(\frac{P^I K}{P^Y Y} \right) \Delta \log K_{t-1} \right] \tag{A.81}$$

where:

$$\Delta \log PR_{t+s} = \Delta \log Y_{t+s} - s_L \sum_{h=1}^{H_K} s_{L_h} \Delta \log L_{h,t} - s_K \Delta \log K_{t+s-1} \tag{A.82}$$

A.3.4 Open Economy

In case of open economy, the household continue to maximize equation (A.1) under the budget constraint (A.2). Therefore, as before, we get equation (A.13). Also in this case, the first five lines

of equation (A.13) equal zero. However, it is not true anymore that in equilibrium $B_t = 0$. Hence, in the case of open economy, equation (A.14) becomes:

$$\begin{aligned} v_t &= v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^L L \widehat{p}_{t+s}^L + \frac{p^K k}{(1+g)(1+n)} \widehat{p}_{t+s}^K - p^C c \widehat{p}_{t+s} + \pi \widehat{\pi}_{t+s} + \frac{rb}{(1+g)(1+n)} \widehat{r}_{t+s}) \right] \\ &+ \lambda \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t-1} + \lambda \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t-1} \end{aligned} \tag{A.83}$$

The budget constraint and the law of motion of capital are unchanged with respect to the benchmark case and therefore equations (A.15) and (A.16) are still valid.

Using these two equations and the steady-state version of the FOC for capital in (A.14) gives us:

$$\begin{aligned} v_t &= v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^C c \widehat{c}_{t+s} + i \widehat{i}_{t+s} - \frac{p^K}{(1+g)(1+n)} k \widehat{k}_{t+s-1} - p^L L \widehat{L}_{t+s} \right] + \lambda \frac{1}{\beta} k \widehat{k}_{t-1} \\ &+ \lambda \sum_{s=0}^{\infty} \beta^s \left[b \widehat{b}_{t+s} - \beta \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t+s} \right] \end{aligned} \tag{A.84}$$

Using the FOC and the trasversality condition for bonds, the last line in the equation above equals zero. But then we obtain equation (A.17) in the benchmark case. Proceeding as before, we get equation (A.19). Define the log-change in domestic absorption as:

$$\Delta \log A_t = s_C \Delta \log C_t + s_I \Delta \log I_t$$

where s_C and s_I are shares out of domestic absorption.

But then all the algebra is the same as before with the only change that GDP is replaced by domestic absorption. Hence:

$$\Delta \log (C_t)^* = \frac{(1-\beta)}{s_C} \left[E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} + \sum_{s=0}^{\infty} \beta^s \Delta E_t \log PR_{t+s} + \frac{1}{\beta} \frac{k}{p^y y} \Delta \log K_{t-1} \right] \tag{A.85}$$

where productivity change is defined as:

$$\Delta \log PR_t = s_C \Delta \log C_t + s_I \Delta \log I_t - s_L \Delta \log L_t - s_K \Delta \log K_{t-1} \tag{A.86}$$

where also s_C and s_I are shares out of domestic absorption.

A.3.5 Summing up

Using the extensions developed in A.3.1-A.3.4 allows us to write equations (35) and (36) in Proposition 1'. A parallel argument can be used to derive the generalization of Proposition 2, Proposition 2'.

Table 1: Log Productivity

Dependent variable: $logPR_t$

CASE 1: Wasteful Government. Lump-Sum Taxes

	Canada	France	Italy	Japan	Spain	UK	USA
log PR(t-1)	0.459	0.967	0.835	1.424	1.174	0.962	0.798
	(0.195)	(0.210)	(0.153)	(0.186)	(0.199)	(0.200)	(0.135)
log PR(t-2)		-0.287		-0.582	-0.393	-0.449	
		(0.204)		(0.187)	(0.195)	(0.179)	
LM1(Prob>chi2)	0.191	0.130	0.444	0.960	0.338	0.450	0.124

CASE 2: Optimal Government. Distortionary Taxes

	Canada	France	Italy	Japan	Spain	UK	USA
log PR(t-1)	0.689	1.107	0.874	1.468	1.388	1.104	0.801
	(0.167)	(0.202)	(0.119)	(0.184)	(0.172)	(0.196)	(0.135)
log PR(t-2)		-0.393		-0.591	-0.623	-0.491	
		(0.197)		(0.186)	(0.172)	(0.182)	
LM1(Prob>chi2)	0.157	0.274	0.166	0.717	0.309	0.820	0.084

Notes: Sample period: 1985-2005 (except Canada: 1985-2004).

Table 2: Annual Average Log Change in Per-Capita Equivalent Consumption

	Wasteful Spending Lump-Sum Taxes	Optimal Spending Lump-Sum Taxes	Wasteful Spending Distortionary Taxes	Optimal Spending Distortionary Taxes
Canada	0.013	0.014	0.021	0.023
France	0.026	0.031	0.026	0.031
Italy	0.018	0.020	0.021	0.023
Japan	0.018	0.025	0.023	0.030
Spain	0.021	0.030	0.030	0.040
UK	0.032	0.036	0.036	0.039
USA	0.025	0.026	0.029	0.030

Notes: Sample period: 1985-2005 (except Canada: 1985-2004).

Table 3: Components of the Annual Log-Change in Per-Capita Equivalent Consumption

	Wasteful Spending Lump-Sum Taxes		Wasteful Spending Distortionary Taxes		Optimal Spending Distortionary Taxes	
	Fraction due to:		Fraction due to:		Fraction due to:	
	TFP	*Capital*	*TFP*	*Capital*	*TFP*	*Capital*
Canada	0.445	0.555	0.658	0.342	0.690	0.310
France	0.830	0.170	0.827	0.173	0.857	0.143
Italy	0.659	0.341	0.707	0.293	0.724	0.276
Japan	0.429	0.571	0.559	0.441	0.661	0.339
Spain	0.512	0.488	0.663	0.337	0.747	0.253
UK	0.816	0.184	0.833	0.167	0.848	0.152
USA	0.830	0.170	0.852	0.148	0.858	0.142

Notes: Sample period: 1985-2005 (except Canada: 1985-2004). TFP includes both expected present value and expectation revision.

Table 4: Annual Average Log-Change in Per-capita Consumption, GDP and Equivalent Consumption

	Consumption	GDP	Equivalent Consumption Opt Gov, Dist Tax
Canada	0.016	0.016	0.023
France	0.016	0.016	0.031
Italy	0.016	0.017	0.023
Japan	0.019	0.018	0.030
Spain	0.027	0.027	0.040
UK	0.024	0.030	0.039
USA	0.020	0.022	0.030

Notes: Sample period: 1985-2005 (except Canada: 1985-2004).

Table 5: Welfare Gap Relative to USA: 1985, 2005 and Average

	Wasteful Spending Lump-Sum Taxes	Wasteful Spending Distortionary Taxes	Optimal Spending Distortionary Taxes
PANEL A: 1985			
Canada	-0.256	-0.294	-0.295
France	-0.069	-0.176	-0.165
Italy	-0.368	-0.420	-0.437
Japan	-0.511	-0.488	-0.471
Spain	-0.327	-0.396	-0.414
UK	-0.096	-0.182	-0.189
USA	0.000	0.000	0.000
PANEL B: 2005			
Canada*	-0.407	-0.455	-0.451
France	-0.078	-0.240	-0.213
Italy	-0.569	-0.641	-0.664
Japan	-0.540	-0.582	-0.526
Spain	-0.396	-0.405	-0.362
UK	0.034	-0.068	-0.059
USA	0.000	0.000	0.000
PANEL C: average 1985-2005			
Canada	-0.328	-0.372	-0.370
France	-0.065	-0.201	-0.181
Italy	-0.445	-0.507	-0.525
Japan	-0.498	-0.505	-0.468
Spain	-0.348	-0.389	-0.376
UK	-0.026	-0.120	-0.120
USA	0.000	0.000	0.000

* Data for Canada end in 2004.

Table 6: Components of Welfare Gap Relative to USA: 1985, 2005 and Average

	Wasteful Spending Lump-Sum Taxes		Wasteful Spending Distortionary Taxes		Optimal Spending Distortionary Taxes	
	Fraction due to:		Fraction due to:		Fraction due to:	
	TFP	*Capital*	*TFP*	*Capital*	*TFP*	*Capital*
PANEL A: 1985						
Canada	0.936	0.064	0.945	0.055	0.945	0.055
France	0.968	0.032	0.988	0.012	0.987	0.013
Italy	1.007	-0.007	1.006	-0.006	1.006	-0.006
Japan	1.091	-0.091	1.096	-0.096	1.099	-0.099
Spain	0.855	0.145	0.880	0.120	0.885	0.115
UK	0.379	0.621	0.672	0.328	0.683	0.317
PANEL B: 2005						
Canada*	0.900	0.100	0.911	0.089	0.910	0.090
France	0.421	0.579	0.811	0.189	0.788	0.212
Italy	0.972	0.028	0.975	0.025	0.976	0.024
Japan	1.065	-0.065	1.060	-0.060	1.067	-0.067
Spain	0.898	0.102	0.900	0.100	0.888	0.112
UK	-	-	-0.079	1.079	-0.233	1.233
PANEL C: average 1985-2005						
Canada	0.915	0.085	0.925	0.075	0.925	0.075
France	0.666	0.334	0.891	0.109	0.879	0.121
Italy	0.999	0.001	0.999	0.001	0.999	0.001
Japan	1.115	-0.115	1.113	-0.113	1.122	-0.122
Spain	0.887	0.113	0.899	0.101	0.895	0.105
UK	-1.513	2.513	0.448	0.552	0.443	0.557

* Data for Canada end in 2004.

Table 7: Per-Capita GDP, Consumption and Equivalent Consumption relative to USA: 1985, 2005 and Average

	Consumption	*GDP*	Equivalent Consumption Opt Gov, Dist Tax
PANEL A: 1985			
Canada	-0.179	-0.106	-0.295
France	-0.285	-0.250	-0.165
Italy	-0.378	-0.287	-0.437
Japan	-0.434	-0.211	-0.471
Spain	-0.624	-0.570	-0.414
UK	-0.306	-0.304	-0.189
USA	0.000	0.000	0.000
PANEL B: 2005			
Canada*	-0.324	-0.177	-0.451
France	-0.401	-0.317	-0.213
Italy	-0.501	-0.370	-0.664
Japan	-0.456	-0.261	-0.526
Spain	-0.527	-0.419	-0.362
UK	-0.190	-0.219	-0.059
USA	0.000	0.000	0.000
PANEL C: average 1985-2005			
Canada	-0.248	-0.163	-0.373
France	-0.338	-0.265	-0.181
Italy	-0.396	-0.278	-0.529
Japan	-0.391	-0.176	-0.468
Spain	-0.537	-0.447	-0.375
UK	-0.220	-0.251	-0.116
USA	0.000	0.000	0.000

* Data for Canada end in 2004.

Figure 1: Within-country welfare comparisons: log equivalent permanent consumption

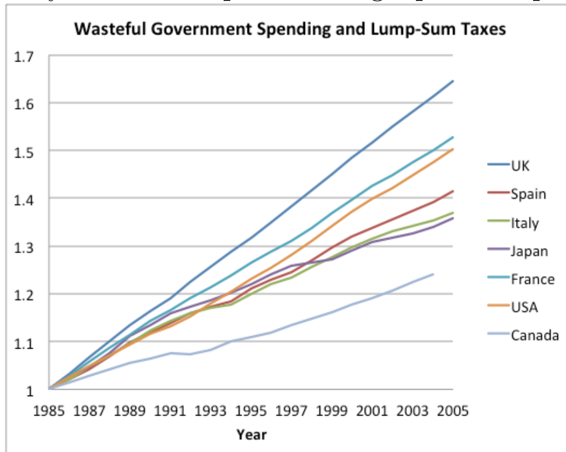

Figure 2: Within-country welfare comparisons: log equivalent permanent consumption.

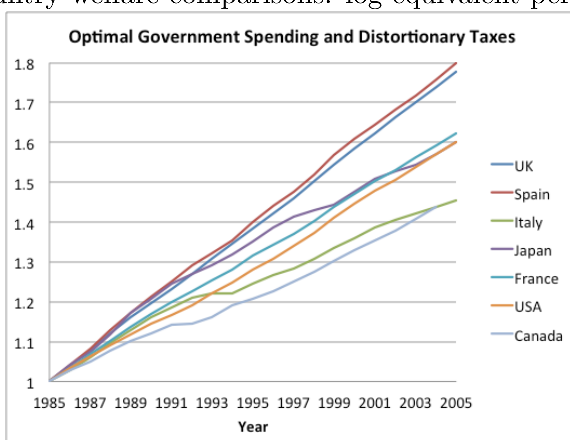

Figure 3: Within-country welfare comparisons: log equivalent permanent consumption (computed using the EU-KLEMS labor service index).

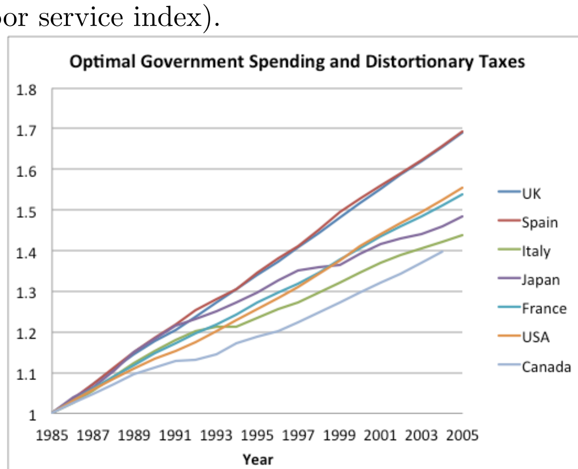

Figure 4: Cross-country welfare comparisons: log equivalent permanent consumption gap (vis-a-vis the US).

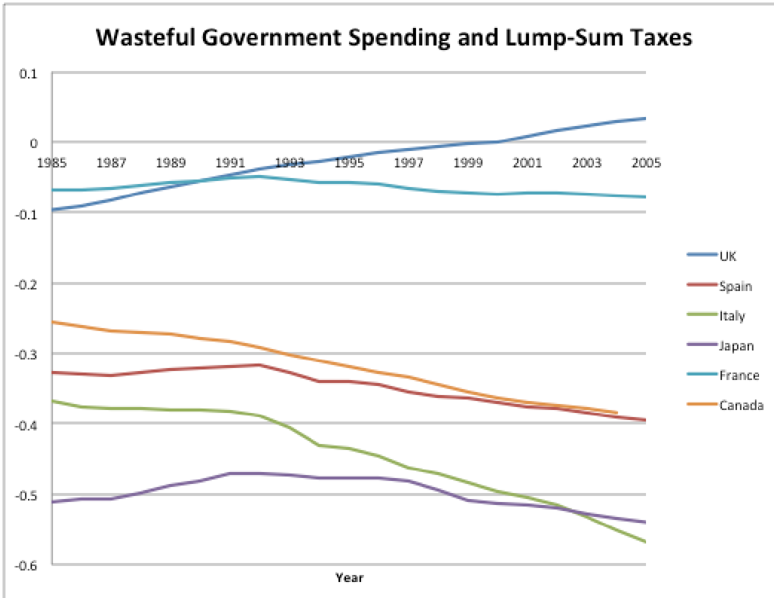

Figure 5: Cross-country welfare comparisons: log equivalent permanent consumption gap (vis-a-vis the US).

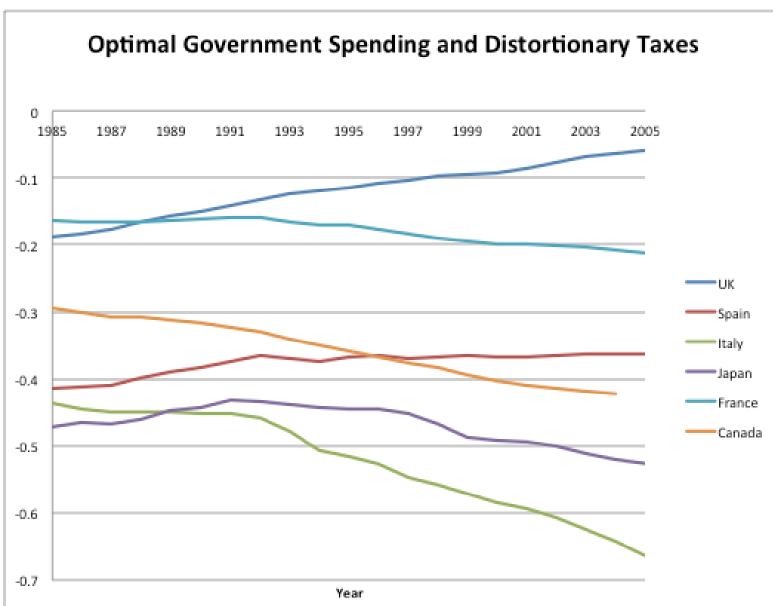

Figure 6: Cross-country welfare comparisons: log equivalent permanent consumption gap (vis-a-vis the US). French preferences.

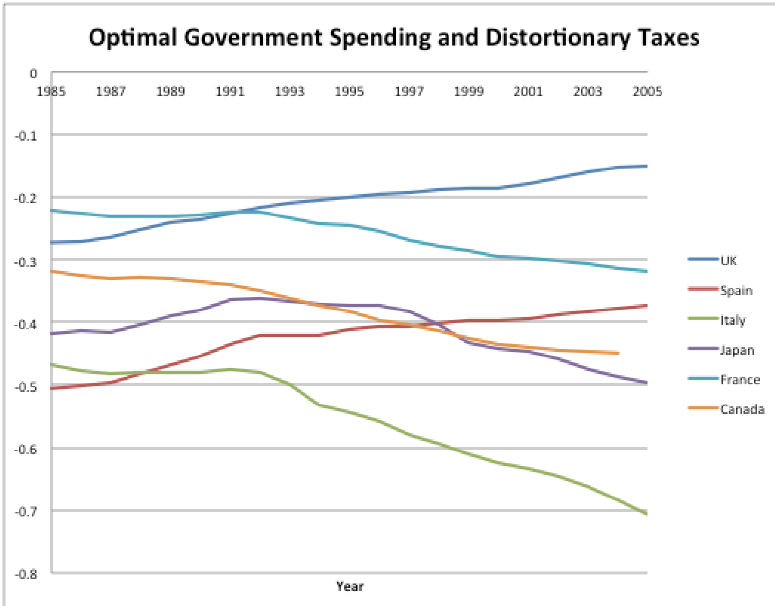

Figure 7: Cross-country welfare comparisons: log equivalent permanent consumption gap (vis-a-vis the US) computed using the EU-KLEMS labor service index.